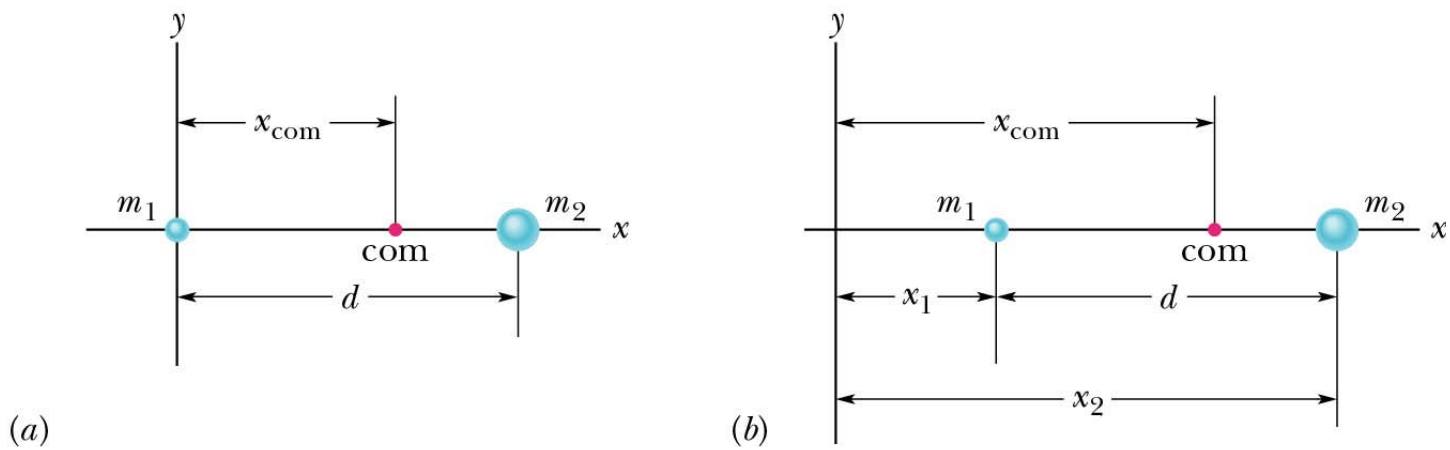
Chapter 09
Center of Mass and Linear Momentum

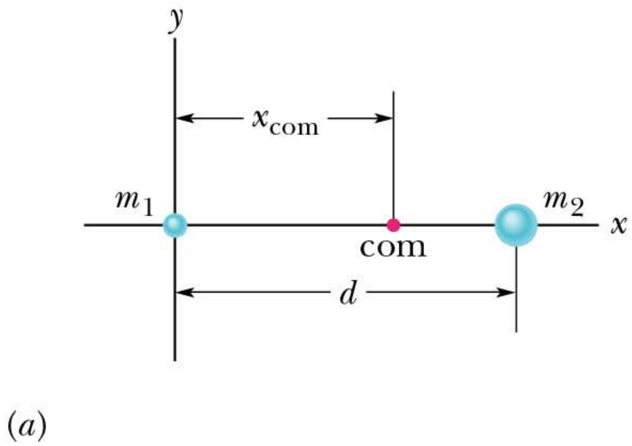
- **Center of mass**: The center of mass of a body or a system of bodies is the point that moves as if all of the mass are concentrated there and all external forces are applied there.
- Note that HRW uses "**com**" but I will use "**c.m.**" because "c.m." is more standard notation for the "center of mass".

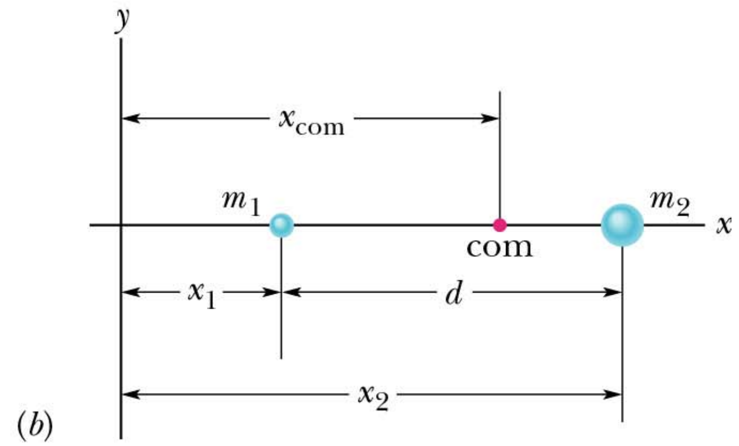


System of Particles

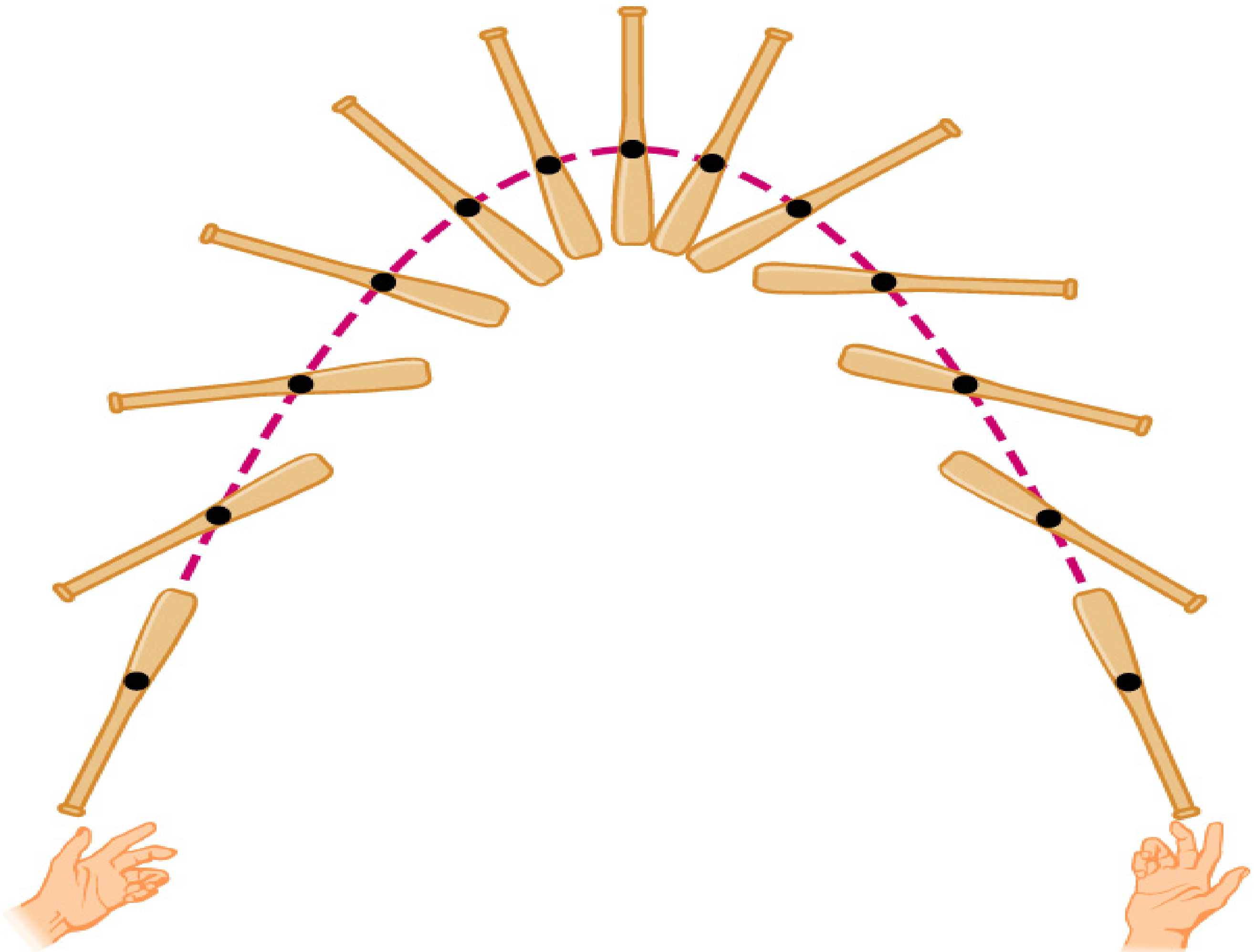
Consider two masses m_1 at $x = x_1$ and m_2 at x_2.

$$x_{c.m.} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$





(a)



(b)

- Center of mass for a system of *n* particles:

$$x_{c.m.} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad y_{c.m.} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i \qquad z_{c.m.} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

- M is just the total mass of the system

$$M = \sum_{i=1}^{n} m_i$$

- Using vectors, we have: $\vec{r}_{c.m.} = x_{c.m.}\hat{i} + y_{c.m.}\hat{j} + z_{c.m.}\hat{k}$

- Therefore:

$$\boxed{\vec{r}_{c.m.} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i}$$

- For a solid body, we can treat it as a continuous distribution of matter dm

$$x_{c.m.} = \frac{1}{M}\int x\,dm \qquad y_{c.m.} = \frac{1}{M}\int y\,dm \qquad z_{c.m.} = \frac{1}{M}\int z\,dm$$

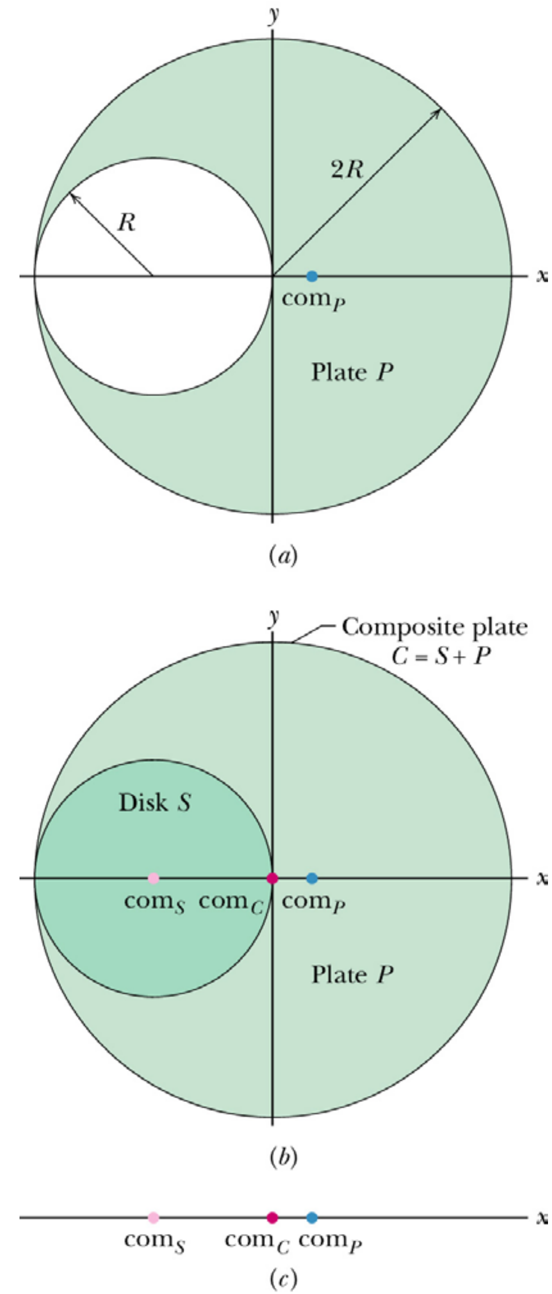
- If the object has uniform density,

  dm = (M/V)dV

$$x_{c.m.} = \frac{1}{V}\int x\,dV \qquad y_{c.m.} = \frac{1}{V}\int y\,dV \qquad z_{c.m.} = \frac{1}{V}\int z\,dV$$

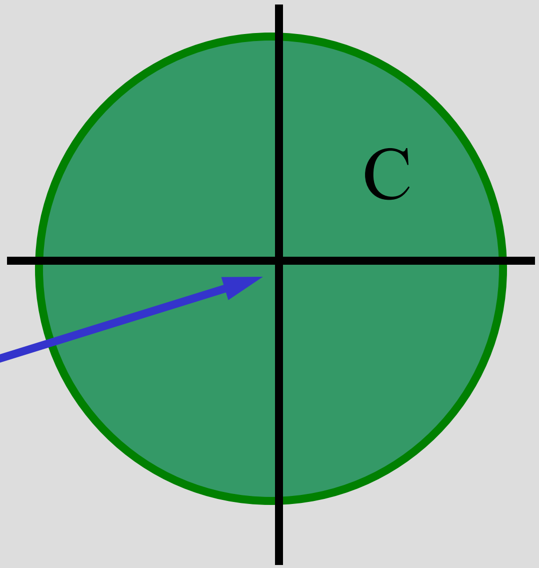
- If an object has a point, a line or a plane of symmetry, the center of mass of such an object then lies at that point, on that line or in that plane.

- Sample 9-2: the figure shows a uniform metal plate P of radius 2R from which a disk of radius R has been stamped out (removed). Using the x-y coordinate system shown, locate the center of mass of the plate.

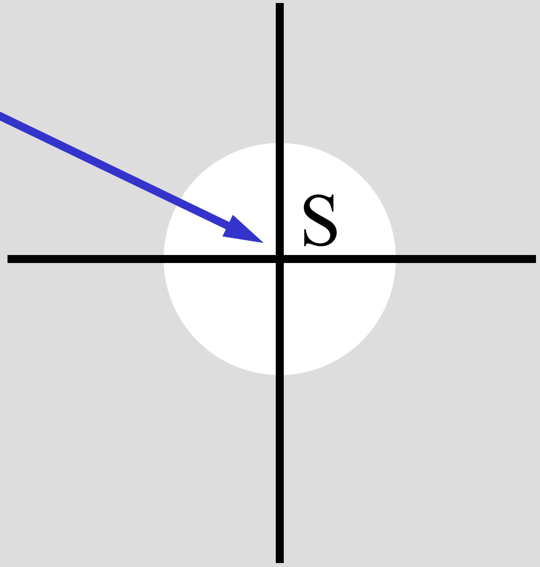


Notice that the object is symmetric about the x-axis, so $y_{c.m.} = 0$. We just need to calculate $x_{c.m.} = 0$.

Technique: Use symmetry as much as possible.

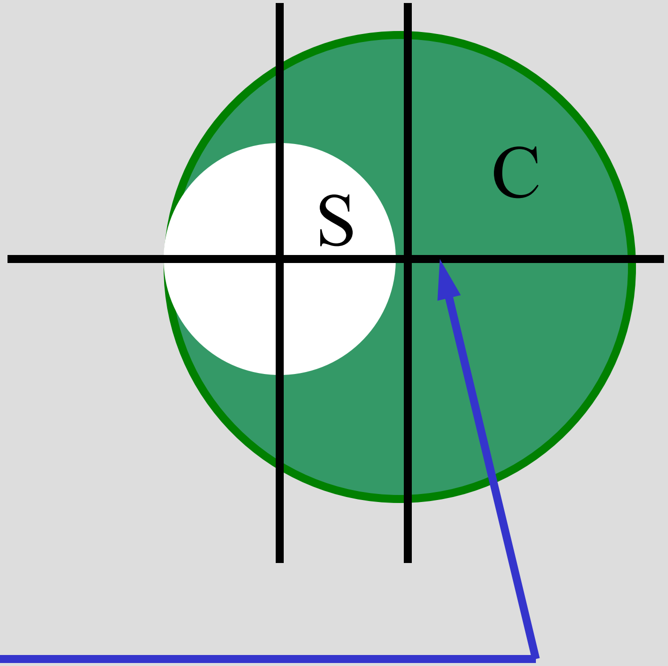


The center of mass of the large disk must be at the center of the disk by symmetry. The same holds true for the small disk.



The hole is as if the smaller disk had "negative" mass to counteract the solid larger mass disk.

Superimpose the two disks. The overlap of the “positive” mass with the “negative” mass will result mathematically in a hole.



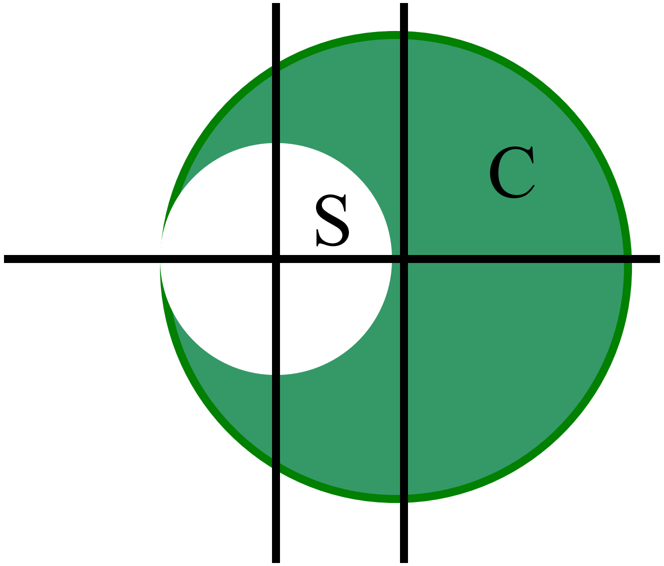
The center of mass will be somewhere over here.

center of mass (c.m. or com) of Plate C: $x_C = 0$

center of mass (c.m. or com) of Disk S: $x_S = -R$

$$x_{c.m.} = \frac{m_C x_C + m_S x_S}{m_C + m_S} = \frac{0 + m_S(-R)}{m_C + m_S} = \left(\frac{-m_S}{m_C + m_S}\right)R$$

Superimpose the two disks. The overlap of the "positive" mass with the "negative" mass will result mathematically in a hole.



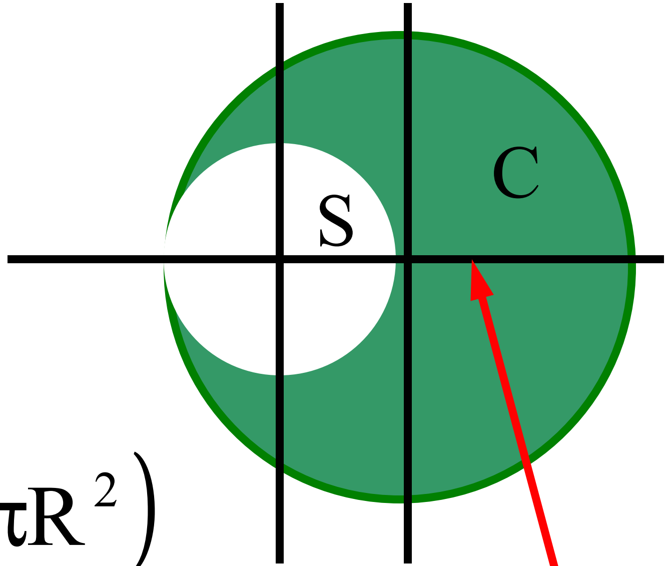
Now we need m_C and m_S. Both disks have the same uniform mass density ρ (but with different "signs"). Thus,

$$m_C = \rho_C V_C = \rho\left(\pi(2R)^2\right) = \rho\left(4\pi R^2\right)$$

Likewise,

$$m_S = \rho_S V_S = -\rho\left(\pi(R)^2\right) = -\rho\left(\pi R^2\right)$$

Superimpose the two disks. The overlap of the "positive" mass with the "negative" mass will result mathematically in a hole.



$$m_C = \rho_C V_C = \rho\left(\pi(2R)^2\right) = \rho\left(4\pi R^2\right)$$

$$m_S = \rho_S V_S = -\rho\left(\pi(R)^2\right) = -\rho\left(\pi R^2\right)$$

$$x_{c.m.} = \frac{m_C x_C + m_S x_S}{m_C + m_S} = \frac{0 + m_S(-R)}{m_C + m_S} = \left(\frac{-m_S}{m_C + m_S}\right)R$$

$$= \left(\frac{-\rho(\pi R^2)}{\rho(4\pi R^2) - \rho(\pi R^2)}\right)R = \frac{1}{3}R$$

- Newton's 2nd law for a system of particles

We know

$$\vec{r}_{c.m.} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i \qquad M\vec{r}_{c.m.} = \sum_{i=1}^{n} m_i \vec{r}_i$$

take derivative with respect to time

$$M\vec{v}_{c.m.} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n$$

take derivative with respect to time again

$$M\vec{a}_{c.m.} = m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots + m_n\vec{a}_n$$

$$M\vec{a}_{c.m.} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n = \vec{F}_{net}$$

Newton's second law for a system of particles

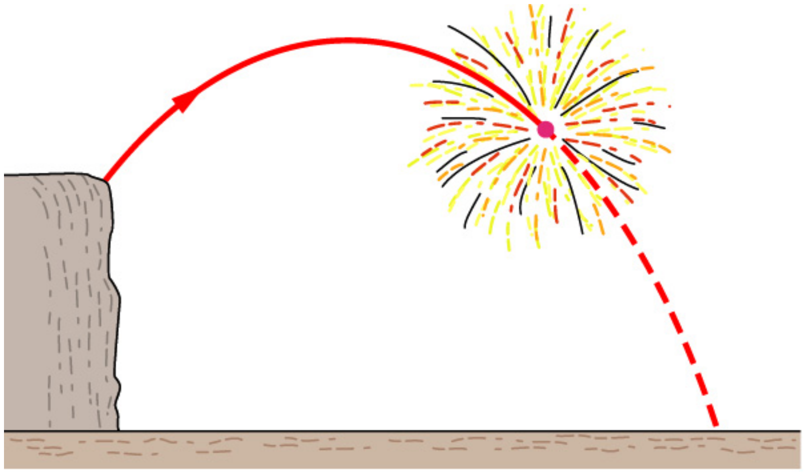
$$F_{net,x} = M a_{c.m.,x} \qquad F_{net,y} = M a_{c.m.,y} \qquad F_{net,z} = M a_{c.m.,z}$$

$$\Rightarrow \qquad \vec{F}_{net} = M \vec{a}_{c.m.}$$

F_{net} is the net force of **all external forces** that act on the system.

M is the total mass of the system.

$a_{c.m.}$ is the acceleration of the center of the mass



Linear Momentum

- The linear momentum of a particle is a vector defined as

$$\vec{p} = m\vec{v}$$

- Newton's second law in terms of momentum

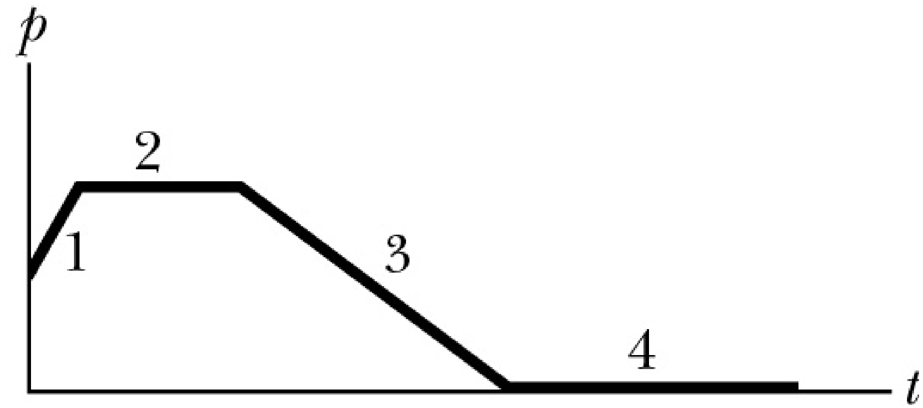
$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} = m\vec{a} = \vec{F}_{net}$$

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

Most of the time the mass doesn't change, so this term is zero. Exceptions are rockets (Monday)

The figure gives the linear momentum versus time for a particle moving along an axis. A force directed along the axis acts on the particle.

(a) Rank the four regions indicated according to the magnitude of the force, greatest first



(b) In which region is the particle slowing?

- Linear momentum of a system of particles

$$\vec{P} = \sum_{i=1}^{n} \vec{p}_i = \sum_{i=1}^{n} m_i \vec{v}_i = M \vec{v}_{c.m.}$$

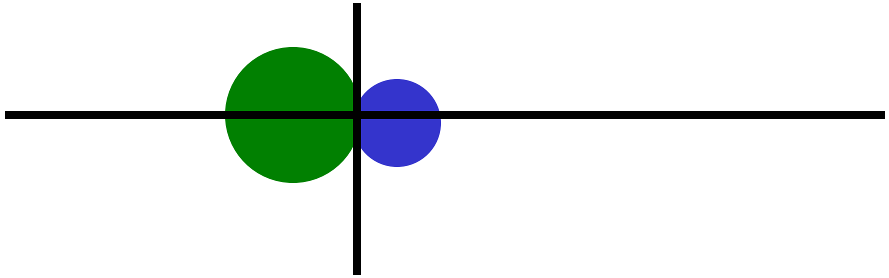
$$\vec{P} = M\vec{v}_{c.m.}$$

- Newton's 2nd law for a system of particles

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{v}_{c.m.}}{dt} = M\vec{a}_{c.m.} = \vec{F}_{net}$$

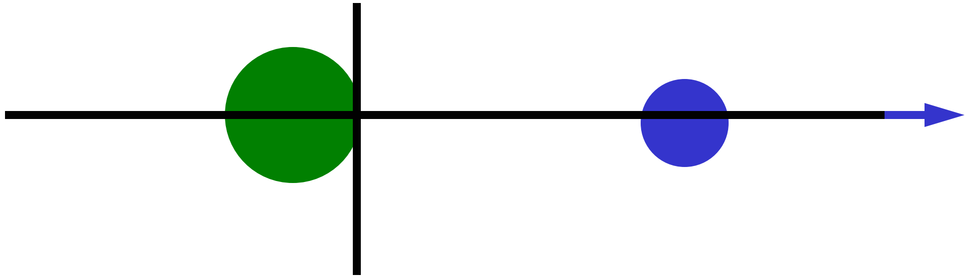
$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

Daily Quiz, February 16, 2005



A helium atom and a hydrogen atom can bind to form the metastable molecule HeH (lifetime of about 1μs). Consider one such molecule at rest in the lab frame at the origin. This molecule then dissociates with the hydrogen atom having momentum $m_p v$ along the +x axis. What happens to the helium atom?

Daily Quiz, February 16, 2005



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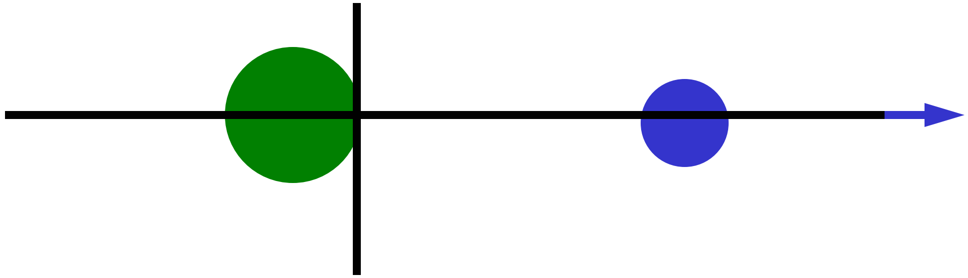

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1) stays at x=0

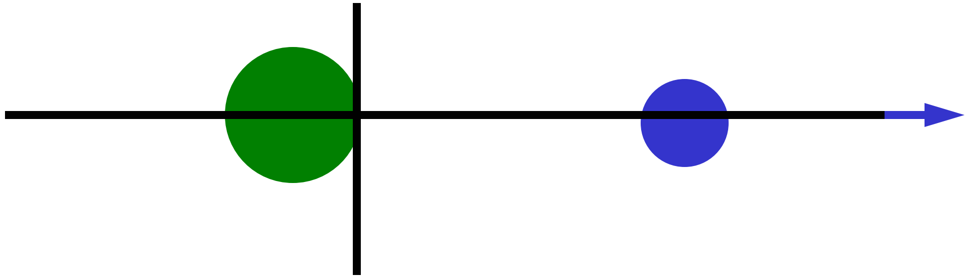
2) goes along +x at speed v

3) goes along –x at speed v

4) goes along –x at speed v/4

5) none of the above

Daily Quiz, February 16, 2005



A helium atom and a hydrogen atom can bind to form the metastable molecule HeH (lifetime of about 1μs). Consider one such molecule at rest in the lab frame at the origin. This molecule then dissociates with the hydrogen atom having momentum $m_p v$ along the +x axis. What happens to the helium ($m_{He} = 4m_p$) atom?

$p_{initial} = 0$ means $p_{final} = 0$

4) goes along –x at speed v/4

$m_p v + m_{He} v_{He} = 0 =$
$m_p v + 4m_p v_{He} = 0$

$\Rightarrow\ v_{He} = -m_p v/4$

Collisions take time!

Even something that seems instantaneous to us takes a finite amount of time to happen.

The collision of a ball with a bat collapses part of the ball.

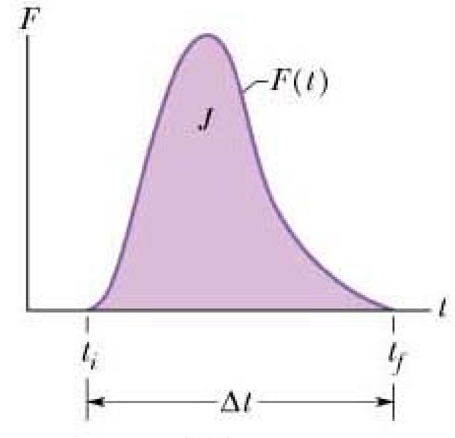
Impulse and Change in Momentum

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \quad \Rightarrow \quad d\vec{p} = \vec{F}dt \quad \Rightarrow \quad \vec{p}_f - \vec{p}_i = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}dt$$

Call this change in momentum the “Impulse” and give it the symbol J.

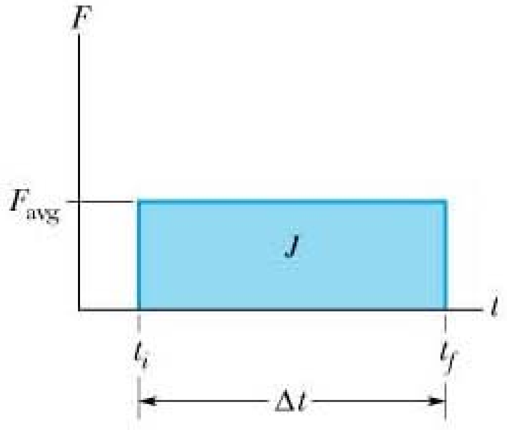
$$\vec{p}_f - \vec{p}_i = \Delta\vec{p} = \vec{J} = \int_{t_i}^{t_f} \vec{F}dt$$

Actual force function versus time.



(a)

Average force versus collision time.



(b)

Impulse and Change in Momentum

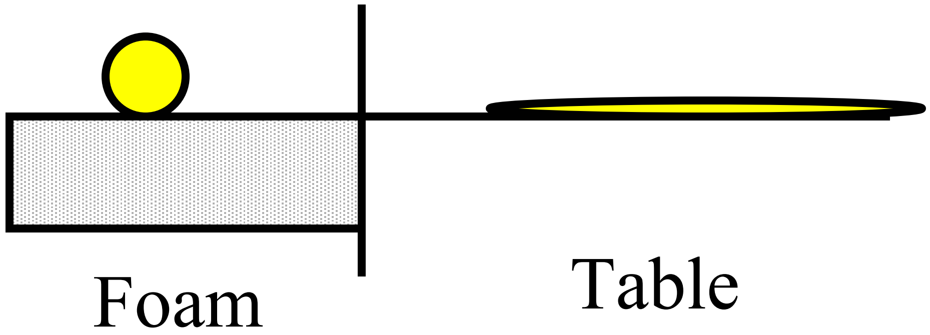
Call this change in momentum the "Impulse" and give it the symbol J.

$$\vec{p}_f - \vec{p}_i = \Delta\vec{p} = \vec{J} = \int_{t_i}^{t_f} \vec{F}dt$$

The instantaneous force is hard (or very difficult) to know in a real collision, but we can use the average force F_{avg}. Thus,

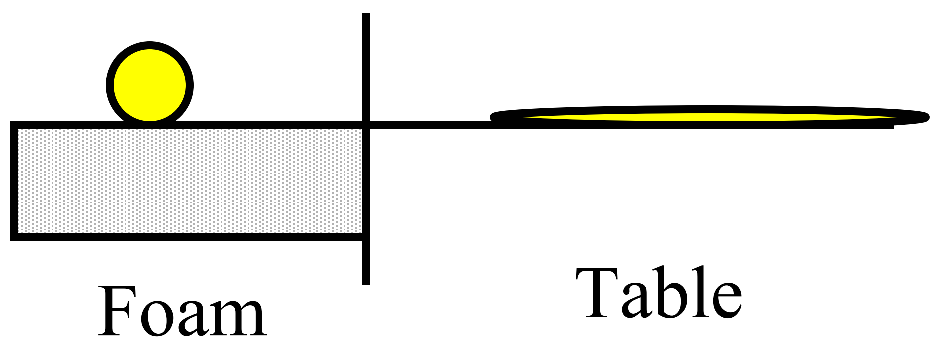
$$J = F_{avg}\Delta t.$$

Daily Quiz, February 18, 2005



An egg was dropped on the table broke, but the egg dropped on the foam pad didn't break. Why didn't this egg break?

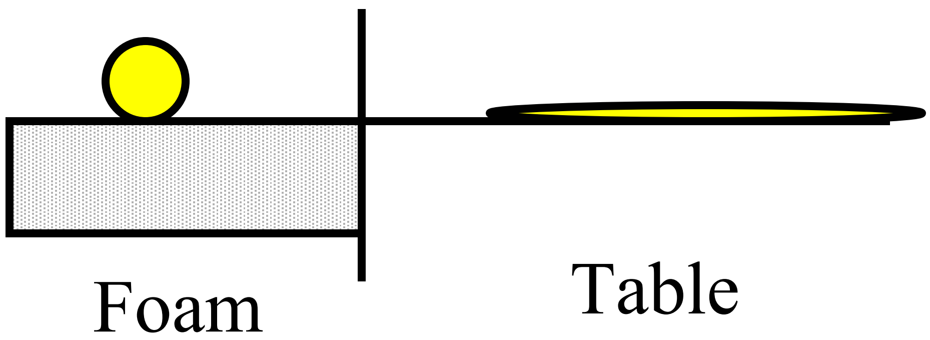
Daily Quiz, February 18, 2005



An egg was dropped on the table broke, but the egg dropped on the foam pad didn't break. Why didn't this egg break?

1) Foam egg's speed was less.
2) Foam egg's change in momentum was less.
3) Foam egg's collision time was greater.
4) What do you mean, it *did* break!   0) none of the above

Daily Quiz, February 18, 2005

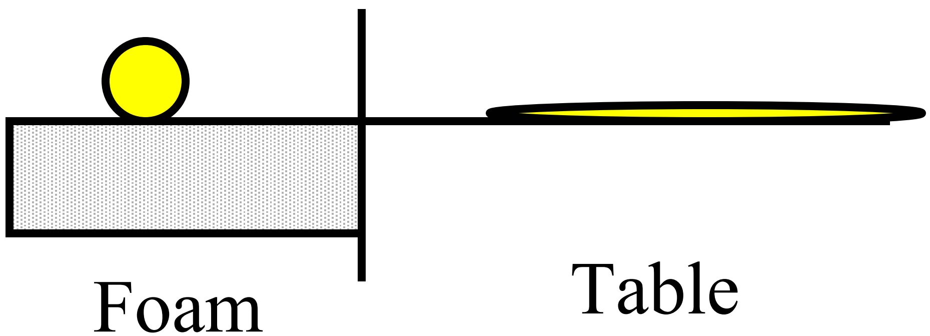


An egg was dropped on the table broke, but the egg dropped on the foam pad didn't break. Why didn't this egg break?

~~1) Foam egg's speed was less.~~

The eggs were dropped from same height, so their speeds were the same. $mgh = 1/2mv^2$

Daily Quiz, February 18, 2005

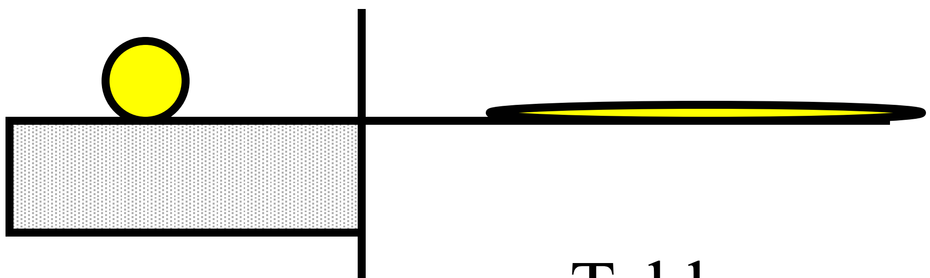


An egg was dropped on the table broke, but the egg dropped on the foam pad didn't break. Why didn't this egg break?

~~2) Foam egg's change in momentum was less.~~

The table egg's momentum stopped at the table, but the foam egg bounced up making its change in momentum greater than the table egg!

Daily Quiz, February 18, 2005



An egg was dropped on the table broke, but the egg dropped on the foam pad didn't break. Why didn't this egg break?

3) Foam egg's collision time was greater.

Since J constant, if Δt is large then F_{avg} will be small.

$$J = F_{avg}\Delta t.$$

Conservation of Linear Momentum

- For a system of particles, if it is both isolated (the net external force acting on the system is zero) and closed (no particles leave or enter the system)….

  If $\Sigma\vec{F} = 0$ then $\dfrac{d\vec{P}}{dt} = 0$

  Therefore $\vec{P} = \text{constant}$ or $\vec{P}_i = \vec{P}_f$

  then the total linear momentum of the system cannot change.

  Law of conservation of linear momentum

- Conservation of linear momentum along a specific direction:

  If $\Sigma F_x = 0$ Then $P_{i,x} = P_{f,x}$

  If $\Sigma F_y = 0$ Then $P_{i,y} = P_{f,y}$

  If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

Linear Momentum

- The linear momentum of a particle is a vector defined as

$$\vec{p} = m\vec{v}$$

- Newton's second law in terms of momentum

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} = m\vec{a} = \vec{F}_{net}$$

Most of the time the mass doesn't change, so this term is zero. Exceptions are rockets (Tuesday)

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

Conservation of Linear Momentum

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  If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

Collisions

In absence of external forces,

Linear momentum is conserved.

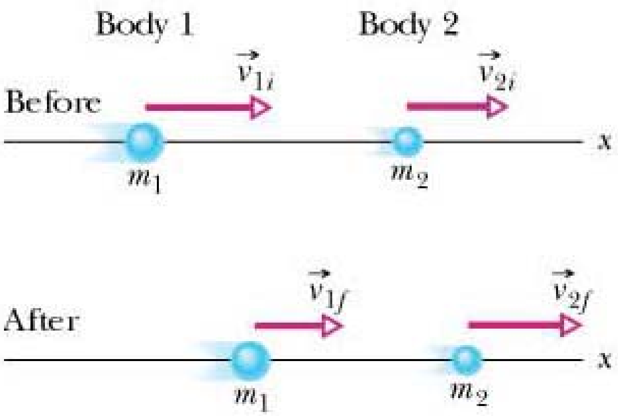
Mechanical energy may or may not be conserved.

Elastic collisions: Mechanical energy is conserved.

Inelastic collisions: Mechanical energy is NOT conserved.

But, **Linear momentum is always conserved.**

Conservation of Linear Momentum

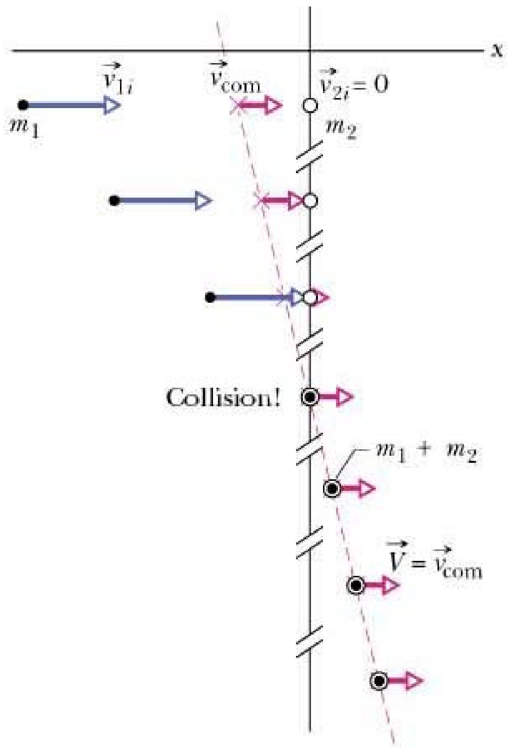


$$\vec{p}_i = \vec{p}_f \quad \Rightarrow$$

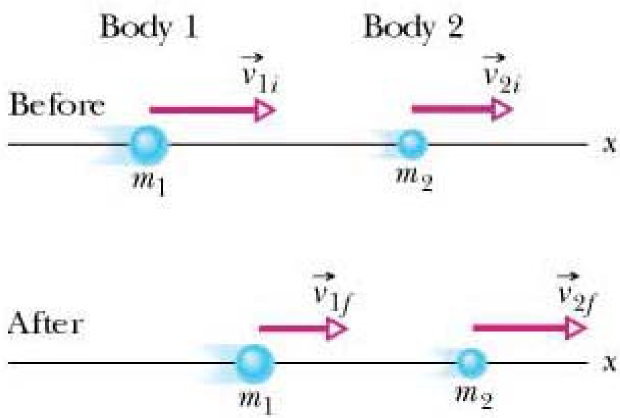
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

Center of Mass motion is constant



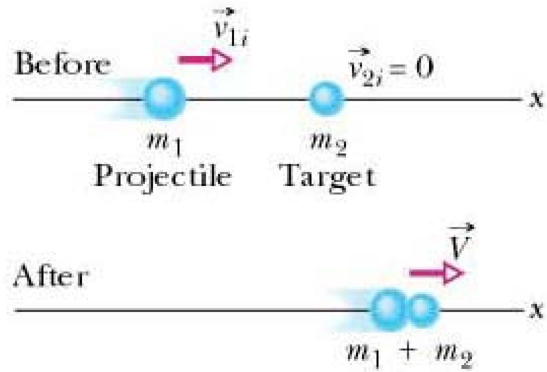
Center of Mass Motion



$$\vec{P} = \vec{p}_{1i} + \vec{p}_{2i} = M\vec{v}_{c.m.} = (m_1 + m_2)\vec{v}_{c.m.}$$

$$\Rightarrow \quad \vec{v}_{c.m.} = \frac{\vec{P}}{m_1 + m_2}$$

Inelastic Collisions



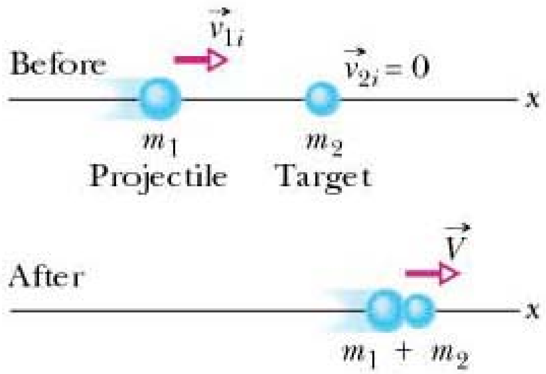
Perfectly inelastic collision: The two masses stick together

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} = (m_1 + m_2)\vec{V}_f$$

0

$$\vec{V}_f = \frac{m_1}{m_1 + m_2} \vec{v}_{1i} \quad (= \vec{v}_{c.m.})$$

Inelastic Collisions



Was the mechanical energy:

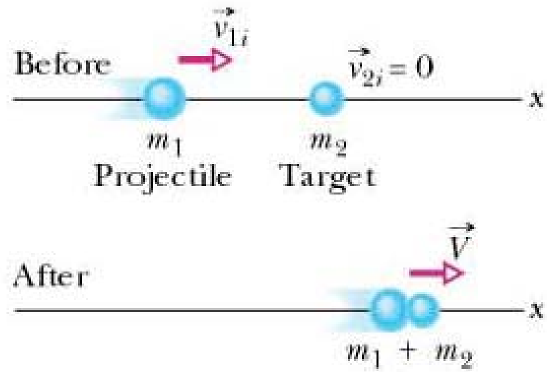
conserved $(E_i = E_f)$;

lost $(E_i > E_f)$; or

gained $(E_i < E_f)$;

in the collision?

Inelastic Collisions



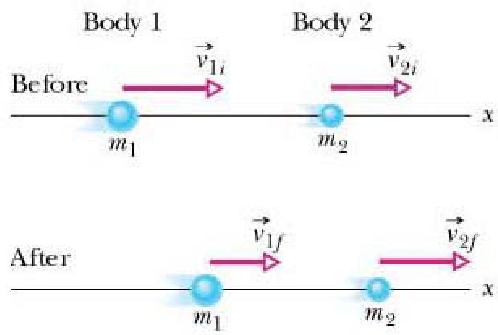
How much mechanical energy was lost in the collision?

$$\frac{1}{2}(m_1 + m_2)\vec{V}_f^2 = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1}{m_1 + m_2}\right)^2 v_{1i}^2 = \frac{1}{2}\left(\frac{m_1^2}{m_1 + m_2}\right)v_{1i}^2 \quad \Rightarrow$$

$$E_{lost} = E_i - E_f = \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}\left(\frac{m_1^2}{m_1 + m_2}\right)v_{1i}^2 = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)v_{1i}^2$$

$$\boxed{E_{lost} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)v_{1i}^2}$$

Elastic Collisions

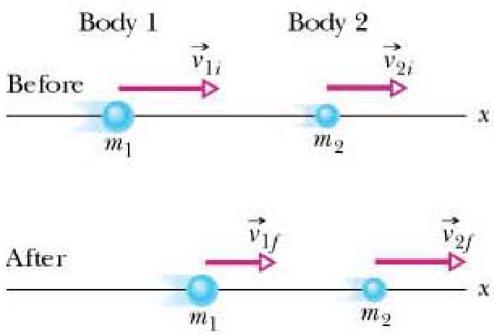


Perfectly elastic collision: Mechanical energy is conserved

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} \quad = \quad m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \qquad (= M\vec{v}_{c.m.})$$

$$E_i = \frac{1}{2}m_1\vec{v}_{1i}^{\,2} + \frac{1}{2}m_2\vec{v}_{2i}^{\,2} \quad = \quad \frac{1}{2}m_1\vec{v}_{1f}^{\,2} + \frac{1}{2}m_2\vec{v}_{2f}^{\,2} = E_f$$

Elastic Collisions



Perfectly elastic collision: Mechanical energy is conserved

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \quad (= M \vec{v}_{c.m.})$$

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} \vec{v}_{1i} + \frac{2m_2}{m_1 + m_2} \vec{v}_{2i}$$

$$\vec{v}_{2f} = \frac{2m_1}{m_1 + m_2} \vec{v}_{1i} + \frac{m_2 - m_1}{m_1 + m_2} \vec{v}_{2i}$$

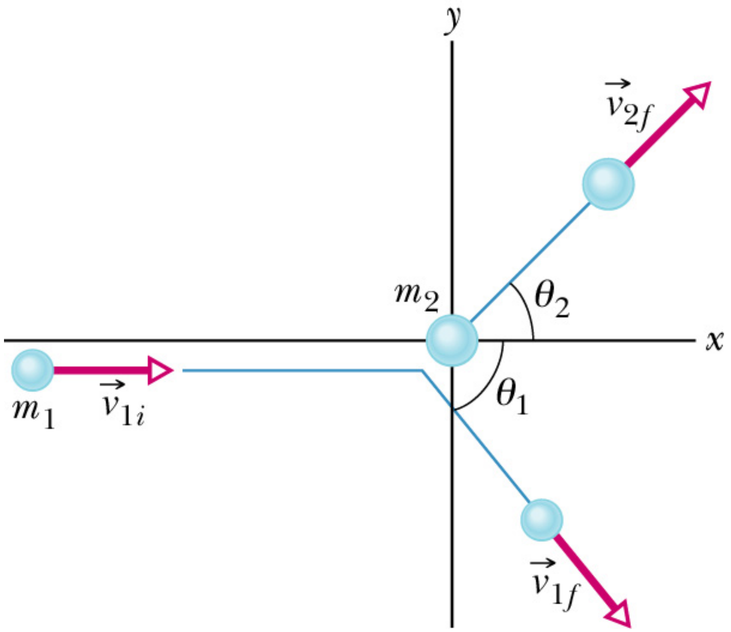
What about two (or more) dimensions?

Simply break the momenta and velocities into their x-, y-, and z-components.

- Inelastic collisions in two dimensions

$$\vec{P}_i = \vec{P}_f$$

$$P_{ix} = P_{fx} \quad P_{iy} = P_{fy}$$



For the case shown here:

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$$

$$0 = m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2$$

Conservation of Linear Momentum

- For a system of particles, if it is both isolated (the net external force acting on the system is zero) and closed (no particles leave or enter the system)….

If $\Sigma \vec{F} = 0$ then $\frac{d\vec{P}}{dt} = 0$

Therefore $\vec{P} = \text{constant}$ or $\vec{P}_i = \vec{P}_f$

then the total linear momentum of the system cannot change.

Law of conservation of linear momentum

Linear Momentum is conserved

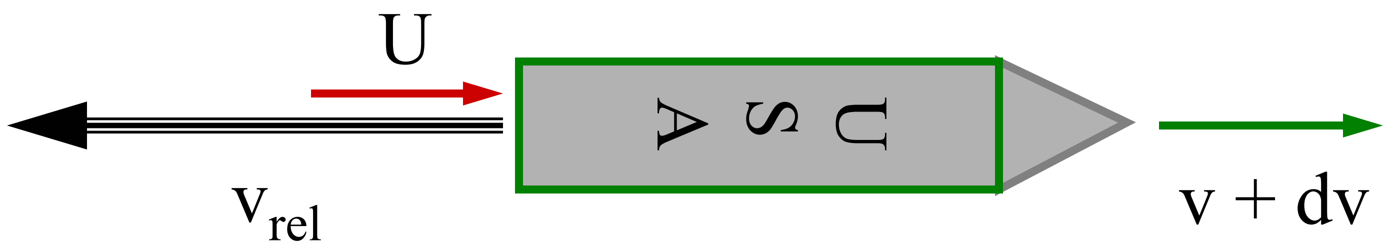
$$\vec{P}_f = \vec{P}_i$$

Let the mass M change (M + dM), which in turn makes the velocity change. Rocket exhausts –dM in time dt at a velocity U relative to our inertial reference frame.

$$Mv = -dM\,U \quad + \quad (M + dM)(v + dv)$$

$$\begin{pmatrix}\text{Velocity of rocket} \\ \text{relative to ref. frame}\end{pmatrix} = \begin{pmatrix}\text{Velocity of rocket} \\ \text{relative to exhaust}\end{pmatrix} + \begin{pmatrix}\text{Velocity of exhaust} \\ \text{relative to ref. frame}\end{pmatrix}$$

$$(v + dv) = v_{rel} \quad + \quad U$$



Linear Momentum is conserved

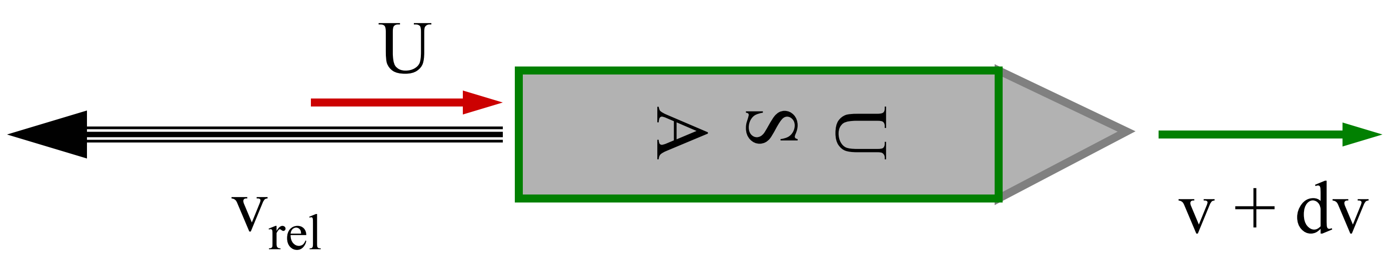
$$\vec{P}_f = \vec{P}_i$$

$$U = (v + dv) - v_{rel}$$

Substitute and divide by dt

$$Mv = -dM\,U \quad + \quad (M + dM)(v + dv)$$

$$\boxed{-\frac{dM}{dt} v_{rel} = M \frac{dv}{dt}}$$



Linear Momentum is conserved

$$\vec{P}_f = \vec{P}_i$$

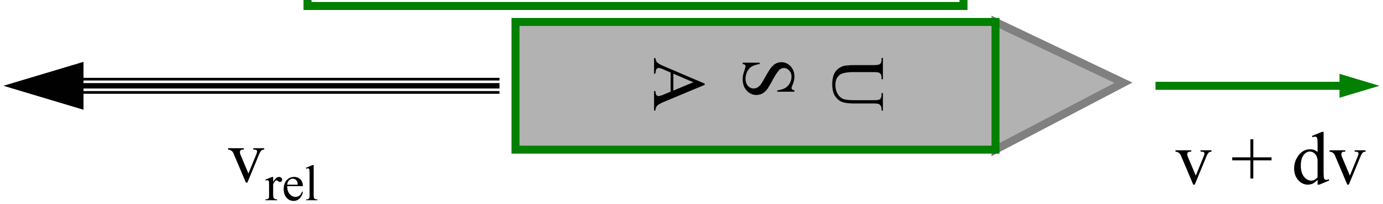
$$U = (v + dv) - v_{rel}$$

Substitute and divide by dt

$$Mv = -dM\,U \quad + \quad (M + dM)(v + dv)$$

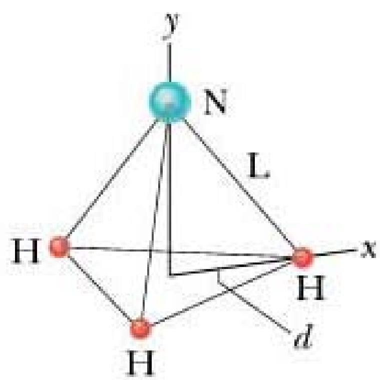
$$dv = -\frac{dM}{M} v_{rel} \Rightarrow \quad \int_{v_i}^{v_f} dv = -v_{rel} \int_{M_i}^{M_f} \frac{dM}{M}$$

$$\Rightarrow \boxed{v_f - v_i = v_{rel} \ln\left(\frac{M_i}{M_f}\right)}$$



Problem 09-05

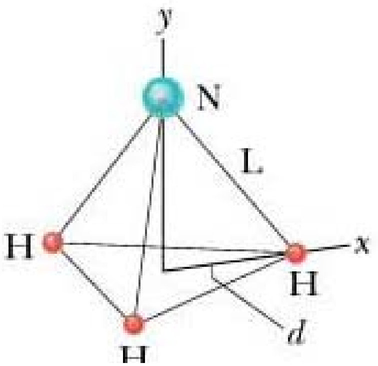
Find the center of mass of the ammonia molecule.



Mass ratio: N/H = 13.9
H to triangle center: $d = 9.40 \times 10^{-11}$m
N to hydrogen: $L = 10.14 \times 10^{-11}$m

Problem 09-05

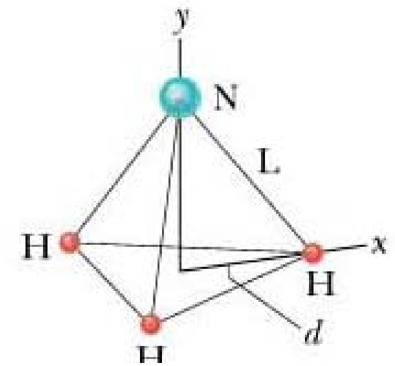
Find the center of mass of the ammonia molecule.



5. (a) By symmetry the center of mass is located on the axis of symmetry of the molecule – the y axis. Therefore $x_{com} = 0$.

Problem 09-05

Find the center of mass of the ammonia molecule.



5. (a) By symmetry the center of mass is located on the axis of symmetry of the molecule – the y axis. Therefore $x_{com} = 0$.

(b) To find y_{com}, we note that $3m_H y_{com} = m_N(y_N - y_{com})$, where y_N is the distance from the nitrogen atom to the plane containing the three hydrogen atoms:

$$y_N = \sqrt{\left(10.14\times10^{-11}\text{ m}\right)^2 - \left(9.4\times10^{-11}\text{ m}\right)^2} = 3.803\times10^{-11}\text{ m}.$$

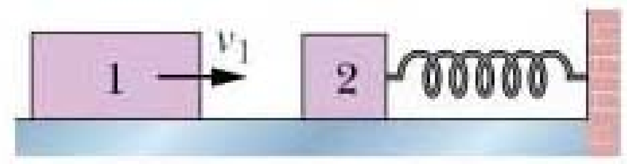
Thus,

$$y_{com} = \frac{m_N y_N}{m_N + 3m_H} = \frac{(14.0067)\left(3.803\times10^{-11}\text{ m}\right)}{14.0067 + 3(1.00797)} = 3.13\times10^{-11}\text{ m}$$

where Appendix F has been used to find the masses.

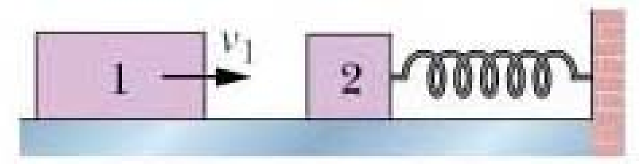
Problem 09-54

Find the distance the spring is compressed. m_1=2.0kg, m_2=1.0kg.



Problem 09-54

Find the distance the spring is compressed. m_1=2.0kg, m_2=1.0kg.



54. We think of this as having two parts: the first is the collision itself – where the blocks "join" so quickly that the 1.0-kg block has not had time to move through any distance yet – and then the subsequent motion of the 3.0 kg system as it compresses the spring to the maximum amount x_m. The first part involves momentum conservation (with $+x$ rightward):

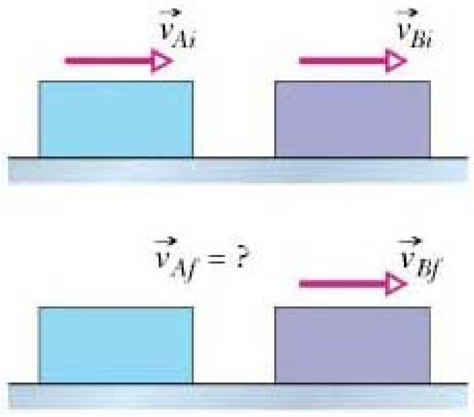
$$m_1v_1 = (m_1+m_2)v \quad \Rightarrow \quad (2.0\text{ kg})(4.0\text{ m/s}) = (3.0\text{ kg})\vec{v}$$

which yields $\vec{v} = 2.7\text{ m/s}$. The second part involves mechanical energy conservation:

$$\frac{1}{2}(3.0\text{ kg})\,(2.7\text{ m/s})^2 = \frac{1}{2}\,(200\text{ N/m})x_m^2$$

which gives the result $x_m = 0.33$ m.

Problem 09-56

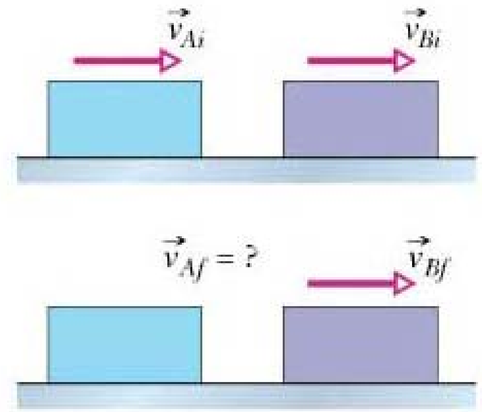


Find the final velocity of A and is the collision elastic?

Mass A: m_A=1.6kg, v_{Ai}=5.5 m/s

Mass B: m_B=2.4kg, v_{Bi}=2.5 m/s, v_{Bf}=4.9 m/s

Problem 09-56



Find the final velocity of A and is the collision elastic?

Mass A: m_A=1.6kg, v_{Ai}=5.5 m/s

Mass B: m_B=2.4kg, v_{Bi}=2.5 m/s, v_{Bf}=4.9 m/s

56. (a) Let m_A be the mass of the block on the left, v_{Ai} be its initial velocity, and v_{Af} be its final velocity. Let m_B be the mass of the block on the right, v_{Bi} be its initial velocity, and v_{Bf} be its final velocity. The momentum of the two-block system is conserved, so

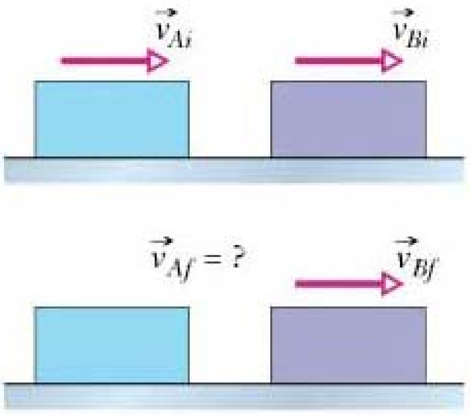
$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

and

$$v_{Af} = \frac{m_A v_{Ai} + m_B v_{Bi} - m_B v_{Bf}}{m_A} = \frac{(1.6)(5.5)+(2.4)(2.5)-(2.4)(4.9)}{1.6} = 1.9 \text{ m/s}.$$

(b) The block continues going to the right after the collision.

Problem 09-56



Find the final velocity of A and is the collision elastic?
Mass A: m_A=1.6kg, v_{Ai}=5.5 m/s
Mass B: m_B=2.4kg, v_{Bi}=2.5 m/s, v_{Bf}=4.9 m/s

(c) To see if the collision is elastic, we compare the total kinetic energy before the collision with the total kinetic energy after the collision. The total kinetic energy before is

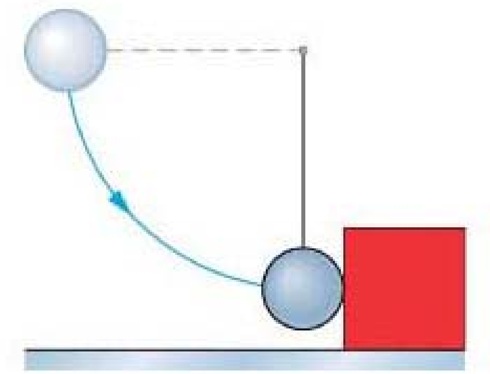
$$K_i = \frac{1}{2}m_A v_{Ai}^2 + \frac{1}{2}m_B v_{Bi}^2 = \frac{1}{2}(1.6)(5.5)^2 + \frac{1}{2}(2.4)(2.5)^2 = 31.7 \text{ J}.$$

The total kinetic energy after is

$$K_f = \frac{1}{2}m_A v_{Af}^2 + \frac{1}{2}m_B v_{Bf}^2 = \frac{1}{2}(1.6)(1.9)^2 + \frac{1}{2}(2.4)(4.9)^2 = 31.7 \text{ J}.$$

Since $K_i = K_f$ the collision is found to be elastic.

Problem 09-60

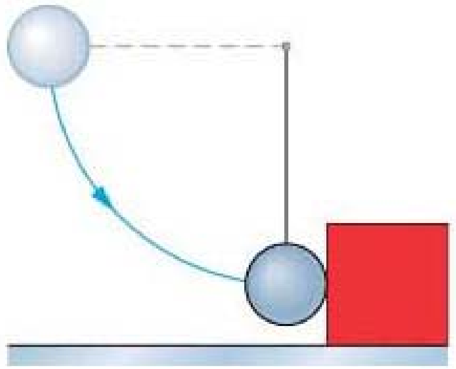


Find the final speeds of the ball and block.

Mass 1 (ball): m_1=0.5kg, h=0.70m

Mass 2 (block): m_2=2.5kg, v_{2i}=0.0 m/s

Problem 09-60



Find the final speeds of the ball and block.
Mass 1 (ball): m_1=0.5kg, h=0.70m
Mass 2 (block): m_2=2.5kg, v_{2i}=0.0 m/s

60. First, we find the speed v of the ball of mass m_1 right before the collision (just as it reaches its lowest point of swing). Mechanical energy conservation (with h = 0.700 m) leads to

$$m_1 gh = \frac{1}{2} m_1 v^2 \Rightarrow v = \sqrt{2gh} = 3.7 \text{ m/s}.$$

(a) We now treat the elastic collision (with SI units) using Eq. 9-67:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v = \frac{0.5 - 2.5}{0.5 + 2.5}(3.7) = -2.47$$

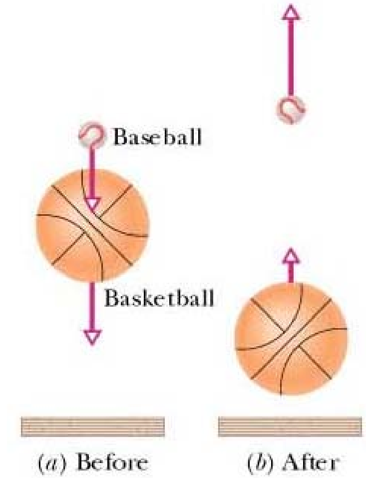
which means the final speed of the ball is 2.47 m/s.

(b) Finally, we use Eq. 9-68 to find the final speed of the block:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v = \frac{2(0.5)}{0.5 + 2.5}(3.7) = 1.23 \text{ m/s}.$$

Problem 09-63

Find the mass m to stop M and the final height of m.
Mass 1 (baseball): $m=?$, $h_{initial}=1.8\text{m}$
Mass 2 (basketball): $M=0.63\text{kg}$, $v_{Mf}=0.0$ m/s

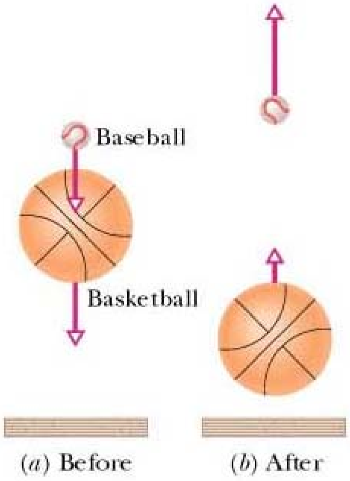


Elastic collisions: Mechanical energy is conserved.

Basketball: $$Mgh = \frac{1}{2}Mv^2 \quad \Rightarrow \quad v = \sqrt{2gh}$$

rebounds upward with same speed -- only reversed direction

Problem 09-63



Find the mass m to stop M and the final height of m.

Mass 1 (baseball): m=?, $h_{initial}$ =1.8m

Mass 2 (basketball): M=0.63kg, v_{Mf}=0.0 m/s

Baseball: $$mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gh}$$

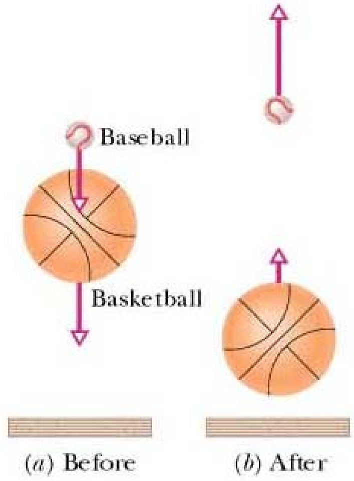
$$v_{Mf} = \frac{M-m}{M+m}v_{Mi} + \frac{2m}{M+m}v_{mi} = \frac{M-m}{M+m}\sqrt{2gh} - \frac{2m}{M+m}\sqrt{2gh}$$

$$= \frac{M-3m}{M+m}\sqrt{2gh} \quad = \quad 0$$

$$\Rightarrow \quad m = M/3 \quad = 0.21\text{kg}$$

Problem 09-63

Find the mass m to stop M and final height of m.
Mass 1 (ball): m_1=0.5kg, h=0.70m
Mass 2 (block): m_2=2.5kg, v_{2i}=0.0 m/s



(b) We use the same equation to find the velocity of the ball of mass *m* after the collision:

$$v_{mf} = -\frac{m-M}{M+m}\sqrt{2gh} + \frac{2M}{M+m}\sqrt{2gh} = \frac{3M-m}{M+m}\sqrt{2gh}$$

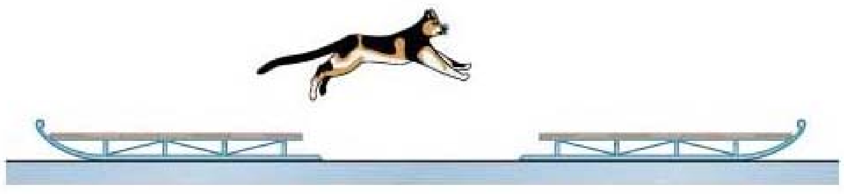
which becomes (upon substituting *M* = 3*m*) $v_{mf} = 2\sqrt{2gh}$. We next use conservation of mechanical energy to find the height *h*′ to which the ball rises. The initial kinetic energy is $\frac{1}{2}mv_{mf}^2$, the initial potential energy is zero, the final kinetic energy is zero, and the final potential energy is *mgh*′. Thus

$$\frac{1}{2}mv_{mf}^2 = mgh' \Rightarrow h' = \frac{v_{mf}^2}{2g} = 4h .$$

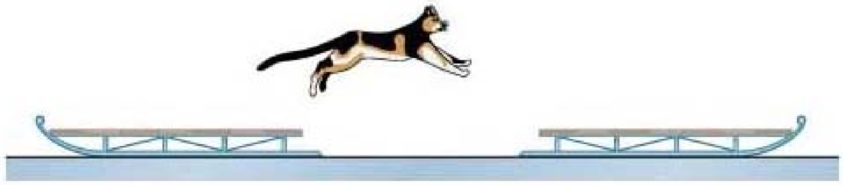
With *h* = 1.8 m, we have *h*’ = 7.2 m.

Problem 09-97

Find the final speeds of the sleds.
Mass 1(sleds): M=22.7kg,
Mass 2 (cat): m=3.63kg, v_i=3.05 m/s



Problem 09-97



97. Let $M = 22.7$ kg and $m = 3.63$ be the mass of the sled and the cat, respectively. Using the principle of momentum conservation, the speed of the first sled after the cat's first jump with a speed of $v_i = 3.05$ m/s is

$$v_{1f} = \frac{mv_i}{M} = 0.488 \text{ m/s}.$$

On the other hand, as the cat lands on the second sled, it sticks to it and the system (sled plus cat) moves forward with a speed

$$v_{2f} = \frac{mv_i}{M+m} = 0.4205 \text{ m/s}.$$

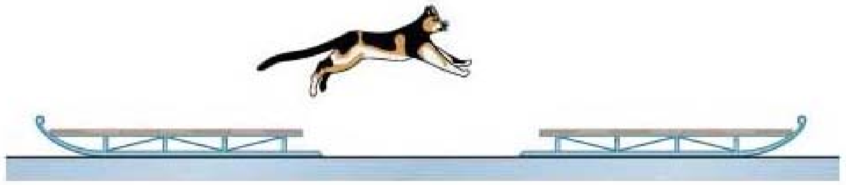
When the cat makes the second jump back to the first sled with a speed v_i, momentum conservation implies

$$Mv_{2ff} = mv_i + (M+m)v_{2f} = mv_i + mv_i = 2mv_i$$

which yields

$$v_{2ff} = \frac{2mv_i}{M} = 0.975 \text{ m/s}.$$

Problem 09-97



After the cat lands on the first sled, the entire system (cat and the sled) again moves together. By momentum conservation, we have

$$(M+m)v_{1ff} = mv_i + Mv_{1f} = mv_i + mv_i = 2mv_i$$

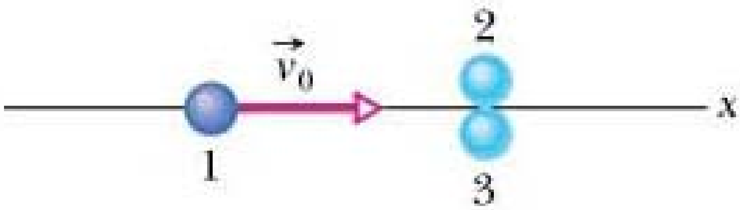
or

$$v_{1ff} = \frac{2mv_i}{M+m} = 0.841 \text{ m/s}.$$

(a) From the above, we conclude that the first sled moves with a speed $v_{1ff} = 0.841$ m/s a after the cat's two jumps.

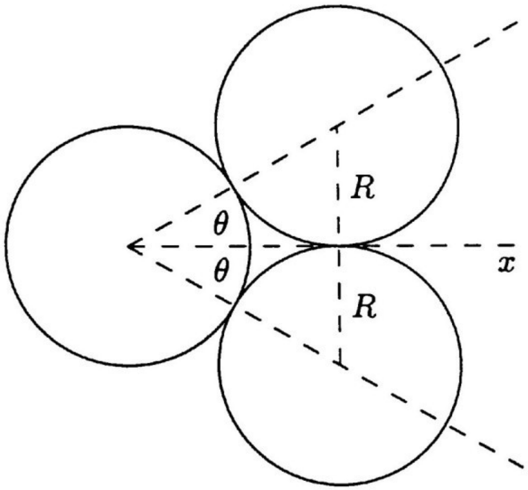
(b) Similarly, the speed of the second sled is $v_{2ff} = 0.975$ m/s.

Problem 09-130

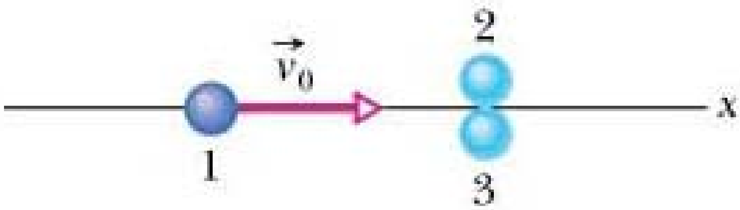


Ball 1 v_o=10.0m/s at contact point of balls 2 and 3. All three balls have mass m. Find the final velocities of all three balls.

130. The diagram below shows the situation as the incident ball (the left-most ball) makes contact with the other two.



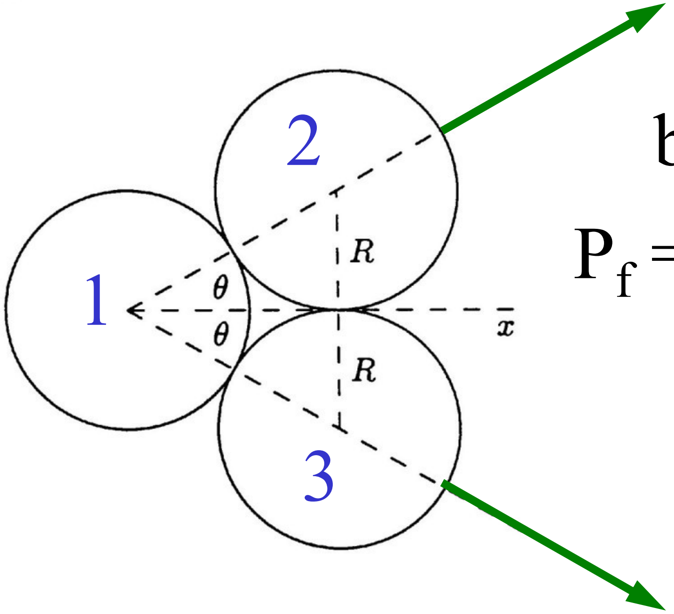
Problem 09-130



Ball 1 v_o=10.0m/s at contact point of balls 2 and 3. All three balls have mass m. Find the final velocities of all three balls.

130. The diagram below shows the situation as the incident ball (the left-most ball) makes contact with the other two.

$P_i = mv_o$

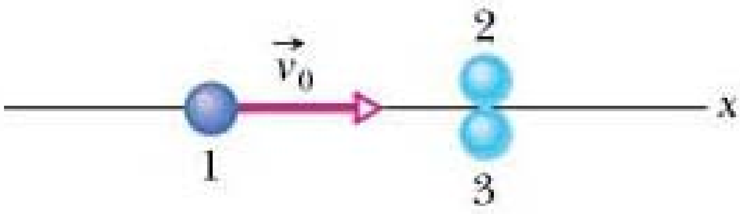


ball1 balls 2 & 3

$P_f = mV + 2mv\cos\theta$

$\theta = 30^o$ since all three balls are identical

Problem 09-130



Ball 1 v_o=10.0m/s at contact point of balls 2 and 3. All three balls have mass m. Find the final velocities of all three balls.

$$mv_0 = mV + 2mv\cos\theta$$

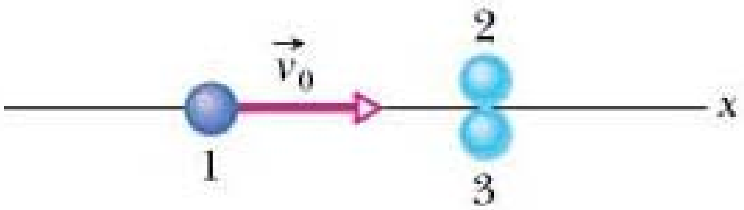
and since the total kinetic energy is conserved,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mV^2 + 2\left(\frac{1}{2}mv^2\right).$$

We know the directions in which the target balls leave the collision so we first eliminate V and solve for v. The momentum equation gives $V = v_0 - 2v\cos\theta$, so

$$V^2 = v_0^2 - 4v_0v\cos\theta + 4v^2\cos^2\theta$$

Problem 09-130



Ball 1 v_o=10.0m/s at contact point of balls 2 and 3. All three balls have mass m. Find the final velocities of all three balls.

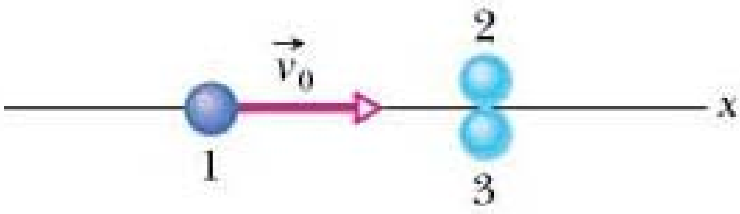
and the energy equation becomes $v_0^2 = v_0^2 - 4v_0 v\cos\theta + 4v^2\cos^2\theta + 2v^2$. Therefore,

$$v = \frac{2v_0\cos\theta}{1+2\cos^2\theta} = \frac{2(10\ \text{m/s})\cos 30°}{1+2\cos^2 30°} = 6.93\ \text{m/s}.$$

(a) The discussion and computation above determines the final speed of ball 2 (as labeled in Fig. 9-83) to be 6.9 m/s.

(b) The direction of ball 2 is at 30° counterclockwise from the +*x* axis.

Problem 09-130



Ball 1 v_o=10.0m/s at contact point of balls 2 and 3. All three balls have mass m. Find the final velocities of all three balls.

(c) Similarly, the final speed of ball 3 is 6.9 m/s.

(d) The direction of ball 3 is at –30° counterclockwise from the +*x* axis.

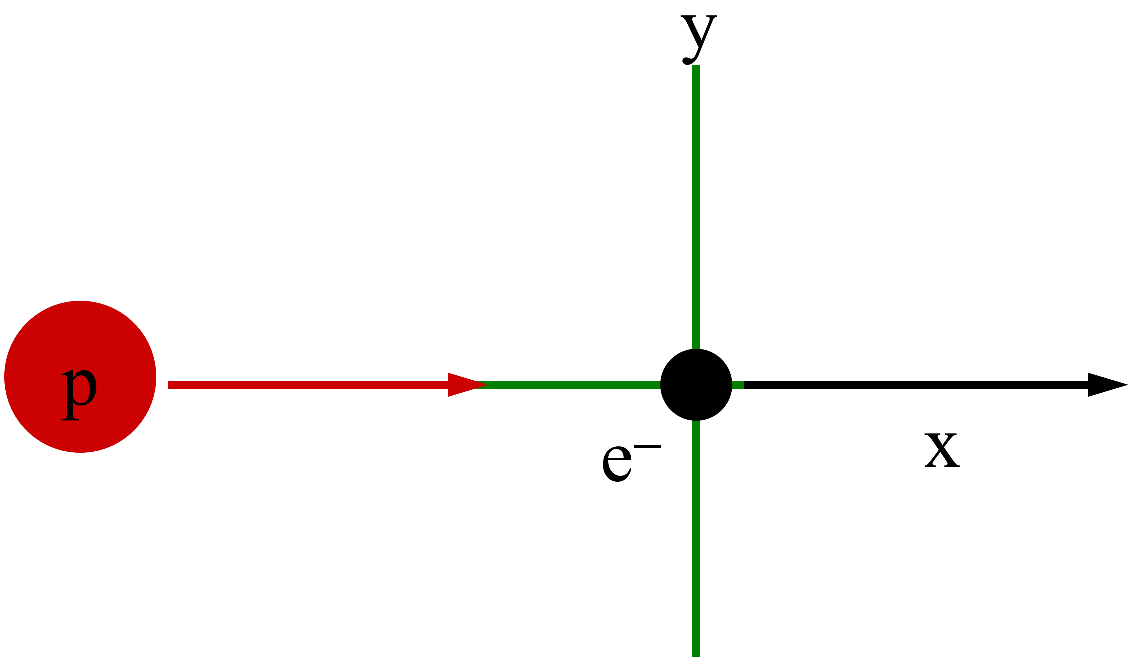
(e) Now we use the momentum equation to find the final velocity of ball 1:

$$V = v_0 - 2v\cos\theta = 10 \text{ m/s} - 2(6.93 \text{ m/s}) \cos 30° = -2.0 \text{ m/s}.$$

So the speed of ball 1 is $|V| = 2.0$ m/s.

(f) The minus sign indicates that it bounces back in the – *x* direction. The angle is –180°.

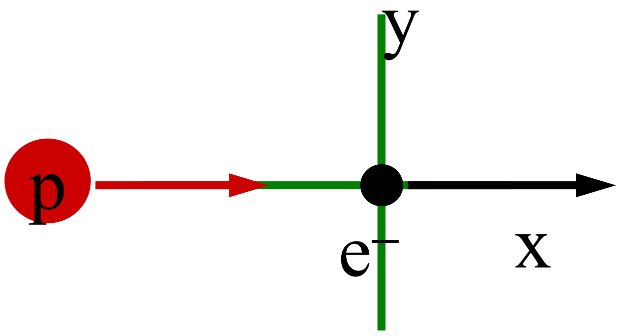
A Quiz



Consider a proton (mass m_p and kinetic energy E_p) colliding head on with an electron (mass m_e) initially at rest. What is the maximum kinetic energy (E_e) that can be delivered to the electron?

A Quiz

Consider a proton (mass $m_p = 1836 m_e$ and kinetic energy E_p) colliding head on with an electron (mass m_e) initially at rest. What is the maximum kinetic energy (E_e) that can be delivered to the electron?



1) $E_e = E_p$ 2) $E_e < E_p$ 3) $E_e > E_p$
4) not enough information

A Quiz

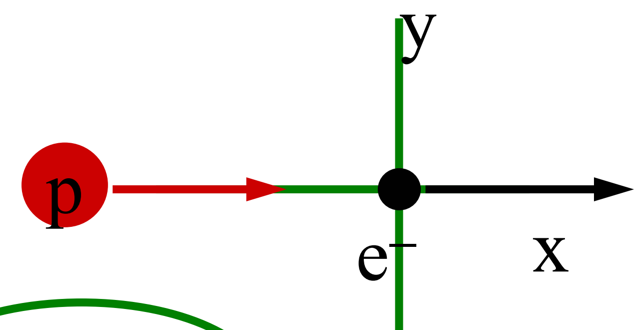
Conservation of Momentum $\quad m_p V_i = m_p V_f + m_p v_f$

Conservation of Energy $\quad \frac{1}{2} m_p V_i^2 = \frac{1}{2} m_p V_f^2 + \frac{1}{2} m_e v_f^2$

$$E_p \equiv \frac{1}{2} m_p V_i^2 \quad \text{and} \quad E_e = \frac{1}{2} m_e v_f^2$$

$$E_e = \frac{4m_e/m_p}{\left(1 + m_e/m_p\right)^2} E_p < E_p$$

Consider a proton (mass $m_p = 1836 m_e$ and kinetic energy E_p) colliding head on with an electron (mass m_e) initially at rest. What is the maximum kinetic energy (E_e) that can be delivered to the electron?



1) $E_e = E_p$ 2) $E_e < E_p$ 3) $E_e > E_p$
4) not enough information