

# Chapter 4: Motion in two and three dimensions

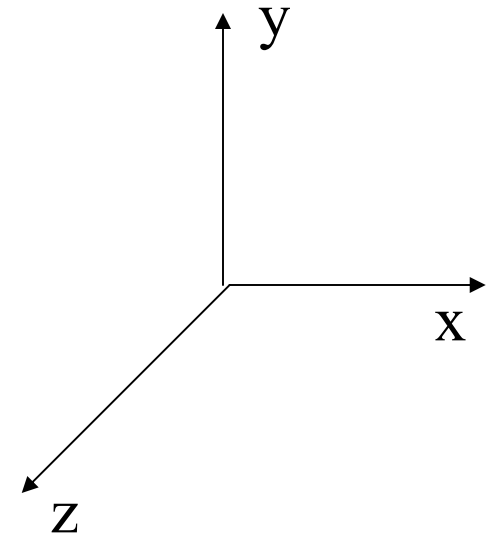
- **Vectors** are needed to describe the 2-D or 3-D motion

- Position vector:

$$\mathbf{r} = x \hat{\mathbf{i}} + y \hat{\mathbf{j}} + z \hat{\mathbf{k}}$$

for example:

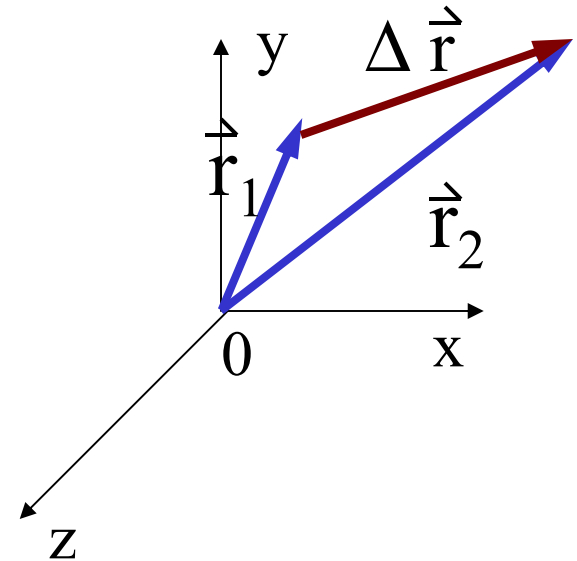
$$\mathbf{r} = (3 \text{ m}) \hat{\mathbf{i}} + (2 \text{ m}) \hat{\mathbf{j}} + (4 \text{ m}) \hat{\mathbf{k}}$$



- Displacement:

from  $\vec{r}_1$  to  $\vec{r}_2$ :  $\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$

$$\Delta \vec{r} = (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j} + (z_2 - z_1) \hat{k}$$



## Example

$$\vec{r} = 1t^2 \hat{i} - (2t + 1) \hat{j} \quad (\text{r in meters and t in seconds})$$

- 1) What is the displacement between  $t = 1\text{ s}$  and  $t = 3\text{ s}$ ?

$$\begin{aligned} \vec{r}_3 &= 3^2 \hat{i} - ((2)(3) + 1) \hat{j} = 9 \hat{i} - 7 \hat{j} \\ \vec{r}_1 &= 1^2 \hat{i} - ((2)(1) + 1) \hat{j} = 1 \hat{i} - 3 \hat{j} \end{aligned} \quad \Rightarrow \quad \Delta \vec{r} = 8 \hat{i} + (-4) \hat{j}$$

- 2) What is the velocity at  $t = 3\text{ s}$ ?

$$\vec{v} = \frac{d\vec{r}}{dt} = 2t \hat{i} - 2 \hat{j} \Big|_{t=3\text{s}} = 6 \hat{i} - 2 \hat{j}$$

- 3) What is the acceleration at  $t = 3\text{ s}$ ?

$$\vec{a} = \frac{d\vec{v}}{dt} = 2 \hat{i} + 0 \hat{j} \Big|_{t=3\text{s}} = 2 \hat{i}$$

# Velocity Vector

- Average velocity between  $t_1$  to  $t_2$

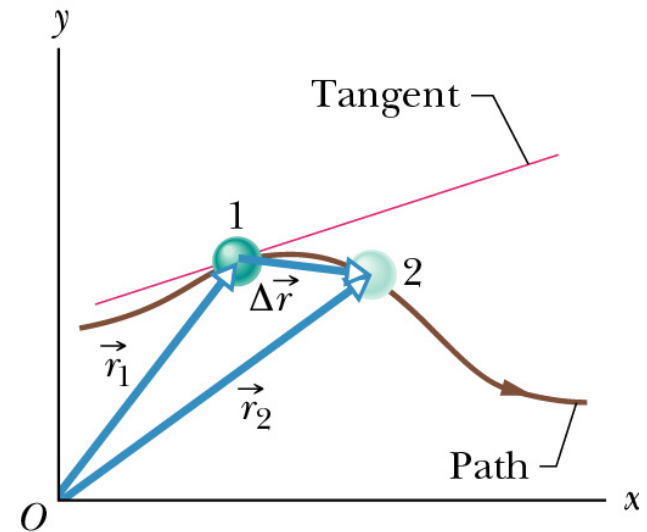
$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

- Instantaneous velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

$$v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \quad v_z = \frac{dz}{dt}$$



$v$  points along the tangent of the path at that position

# Acceleration vector

- Average acceleration between  $t_1$  to  $t_2$

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

- Instantaneous acceleration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$a_x = \frac{dv_x}{dt} \quad a_y = \frac{dv_y}{dt} \quad a_z = \frac{dv_z}{dt}$$

## All 3-D motion can be broken into 3 1-D motions along the 3 axes.

$v_{o_x}$  is the projection of  $\vec{v}_o$  on the x – axis.

Likewise for  $v_{o_y}$ ,  $v_{o_z}$  and the other vector quantities.

$$\begin{aligned}\vec{r} &= x \hat{i} + y \hat{j} + z \hat{k} \\ \vec{v} &= v_x \hat{i} + v_y \hat{j} + v_z \hat{k} \\ \vec{a} &= a_x \hat{i} + a_y \hat{j} + a_z \hat{k}\end{aligned}$$

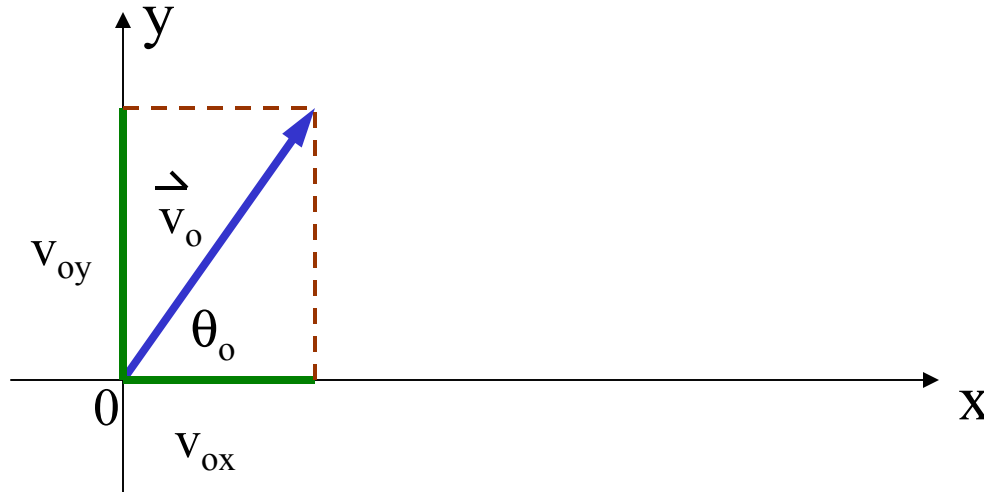
$$x = x_o + v_{o_x} t + \frac{1}{2} a_x t^2$$

$$y = y_o + v_{o_y} t + \frac{1}{2} a_y t^2$$

$$z = z_o + v_{o_z} t + \frac{1}{2} a_z t^2$$

Three 1-D motions

# Projectile Motion in 2-D



- Initial velocity:

$$\vec{v}_0 = v_{0x} \hat{i} + v_{0y} \hat{j}$$

$$v_{0x} = v_0 \cos \theta_0, \quad v_{0y} = v_0 \sin \theta_0$$

# Acceleration Components

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

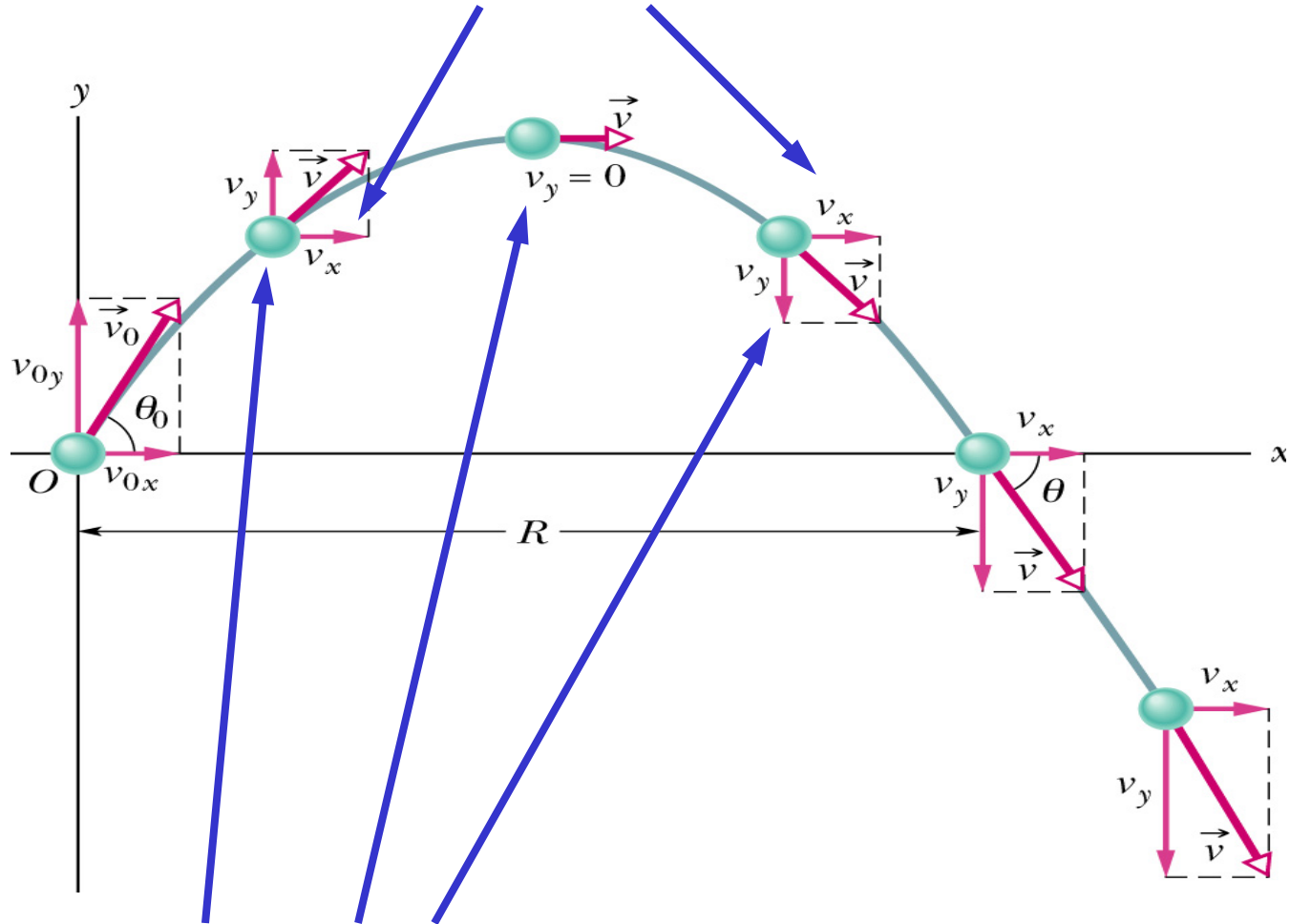
$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}; \quad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}; \quad a_z = \frac{dv_z}{dt} = \frac{d^2z}{dt^2}$$

Consider gravity,  $g$ . Then,  $a_x = a_z = 0$ ;  $a_y = -g$



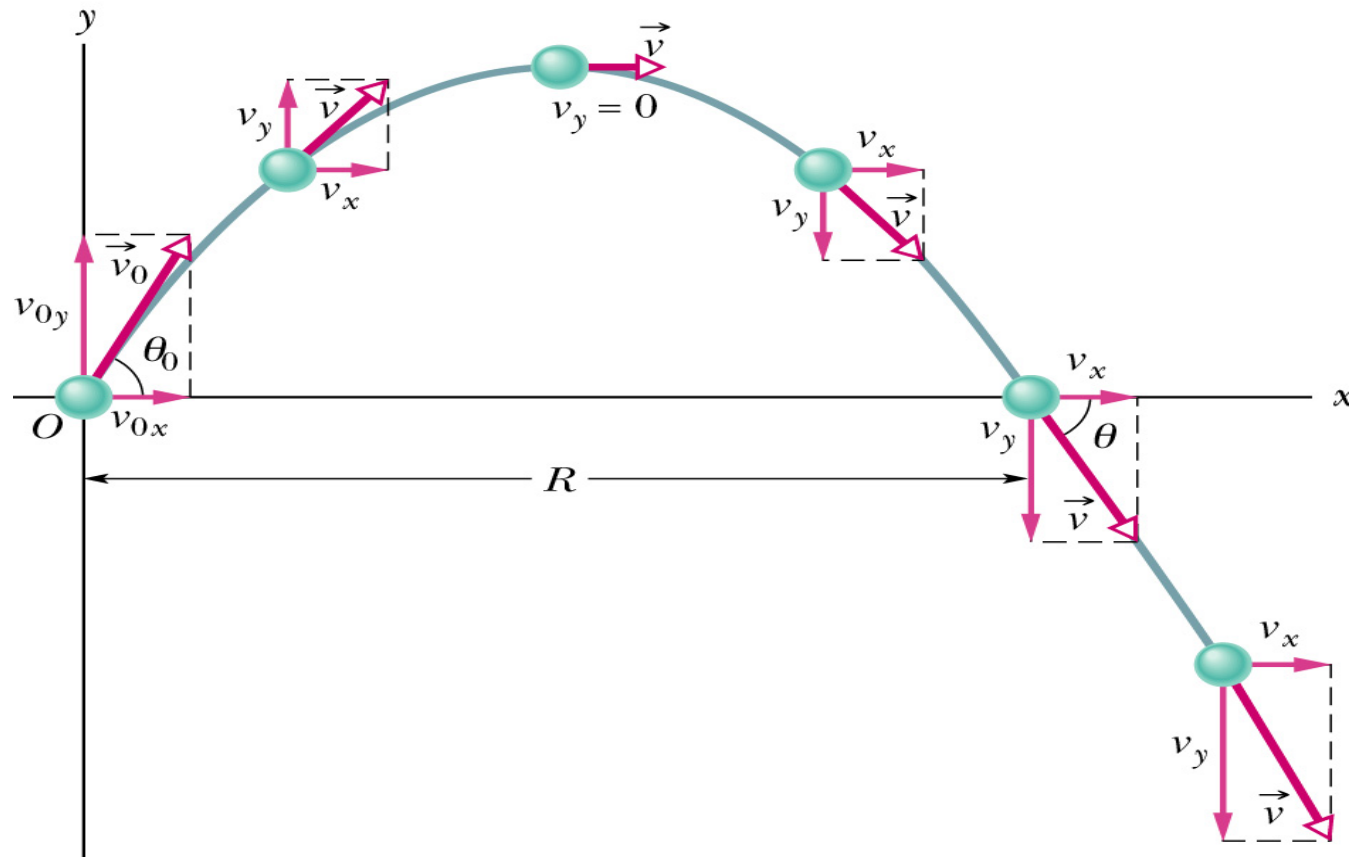
# Projectile motion in 2-D

$a_x = 0$ , which means  $v_x$  is constant



$a_y = -g$ , which then changes  $v_y$  magnitude and direction.

# Projectile motion in 2-D



- The horizontal motion and the vertical motion are independent of each other

- **Horizontal motion:**

Motion with constant velocity

$$v_x = v_{0x} = v_0 \cos \theta_0$$

$$x - x_0 = (v_0 \cos \theta_0) t$$

- **Vertical motion:**

Motion of free-falling object

$$v_y = v_{0y} + a_y t = (v_0 \sin \theta_0) - g t$$

$$y - y_0 = v_{0y} t + \frac{1}{2} a t^2 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2$$

assume the upward direction is positive

# A Quiz



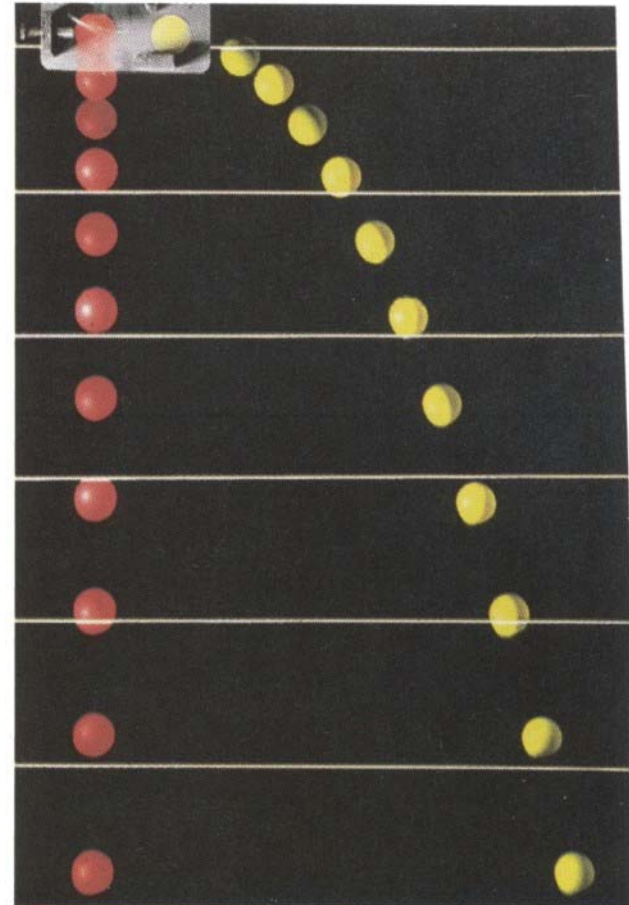
Two identical balls are released at the same time and at the same height. One ball is dropped while the other is launched horizontally. Which ball will hit the floor first? Neglect air resistance.

- 1) both hit at the same time
- 2) launched (yellow ball)
- 3) dropped (red ball)
- 4) more information needed
- 0) none of the above.

# A Quiz

Two identical balls are released at the same time and at the same height. One ball is dropped while the other is launched horizontally. Which ball will hit the floor first? Neglect air resistance.

1) both hit at the same time



**Fig. 4-11** One ball is released from rest at the same instant that another ball is shot horizontally to the right. Their vertical motions are identical.

# Projectile motion analyzed

assume  $x_0 = 0$  and  $y_0 = 0$ ,

$$x = (v_0 \cos \theta_0) t \quad (1)$$

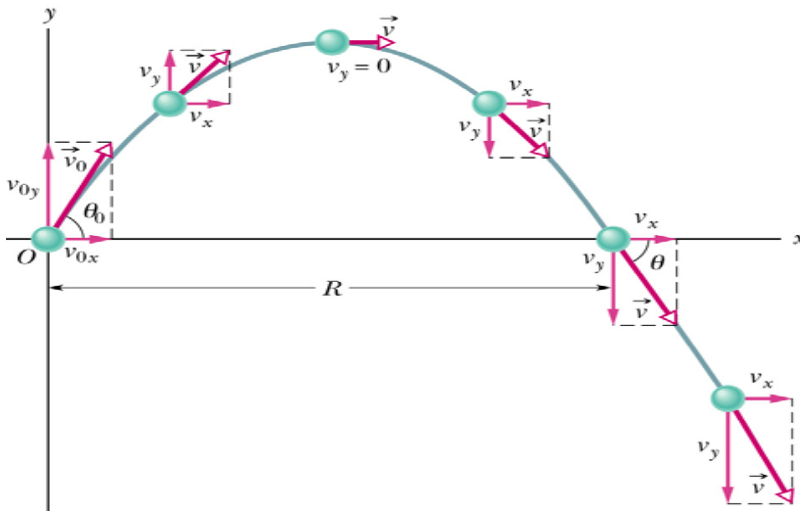
$$y = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 \quad (2)$$

## The equation of the path

From (1):  $t = x / (v_0 \cos \theta_0)$

plug into (2):  $y = (\tan \theta_0) x - [g/2(v_0 \cos \theta_0)^2] x^2$

$$y = a x - b x^2 \quad \text{Parabolic}$$



$$x - x_0 = ( v_0 \cos \theta_0 ) t \quad (1)$$

$$y - y_0 = ( v_0 \sin \theta_0 ) t - \frac{1}{2} g t^2 \quad (2)$$

### **The horizontal range R**

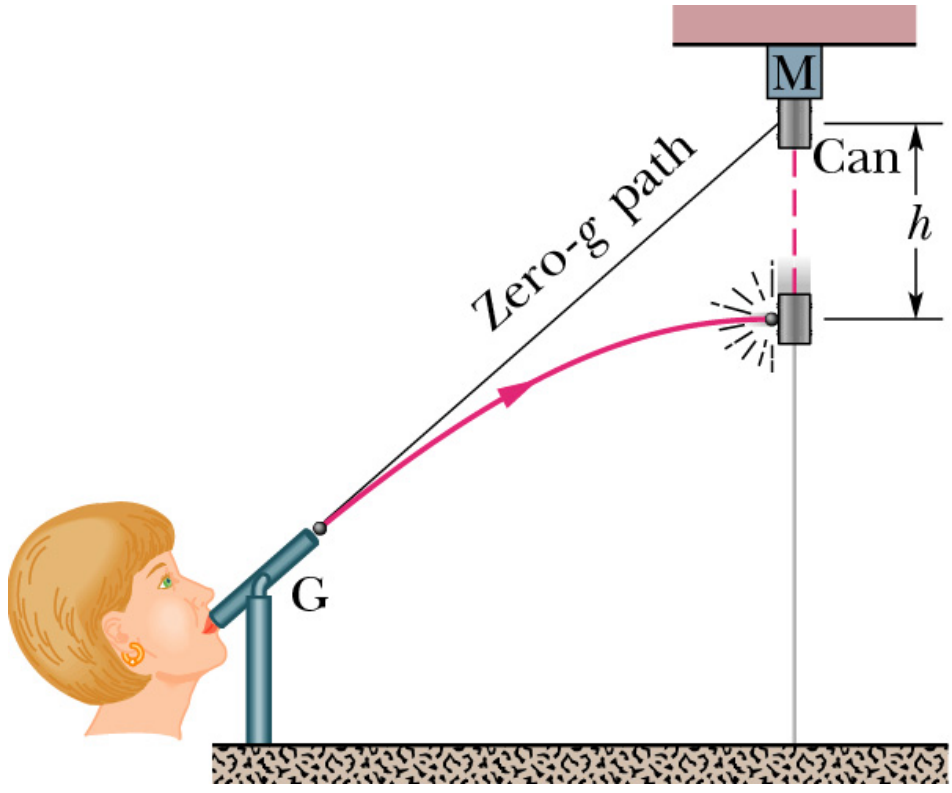
$$R = x - x_0 \text{ when } y - y_0 = 0$$

$$\text{From (2): } 0 = ( v_0 \sin \theta_0 ) t - \frac{1}{2} g t^2$$

$$t = 2 ( v_0 \sin \theta_0 ) / g \quad \text{or} \quad t = 0$$

$$R = ( v_0 \cos \theta_0 ) 2 ( v_0 \sin \theta_0 ) / g = ( v_0^2 / g ) \sin 2 \theta_0$$

When  $\theta_0 = 45^\circ$ ,  $R = v_0^2 / g$  maximum





Check Point 5-5: A fly ball is hit to the outfield.. During its flight(ignore the effect of the air). What happens to its

(a) horizontal components of velocity?

(b) vertical components of velocity?

What are the (c)horizontal and (d)vertical components of its acceleration during its ascent and its descent, and at the topmost point of its flight?

## Projectile motion:

- 1) Select a coordinate system
- 2) Resolve the initial  $v$  vector into  $x$  and  $y$  components
- 3) Treat the horizontal motion and the vertical motion independently
- 4) Analyze the horizontal motion of the projectile as a particle under constant velocity
- 5) Analyze the vertical motion of the projectile as a particle under constant acceleration (  $a = -g$  )

# Uniform circular motion

- Period of revolution (period)

$$T = 2\pi r/v$$

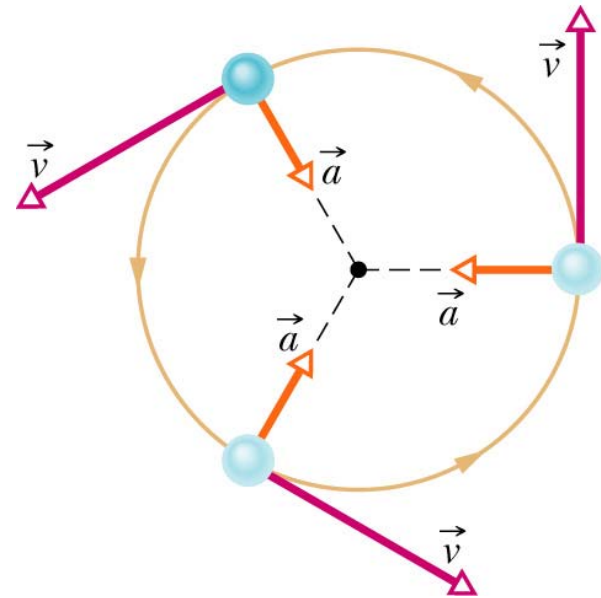
- Centripetal acceleration

magnitude:  $a = v^2 / r$

direction: radially inward

- $v$  and  $a$ : constant magnitude

but vary continuous in direction

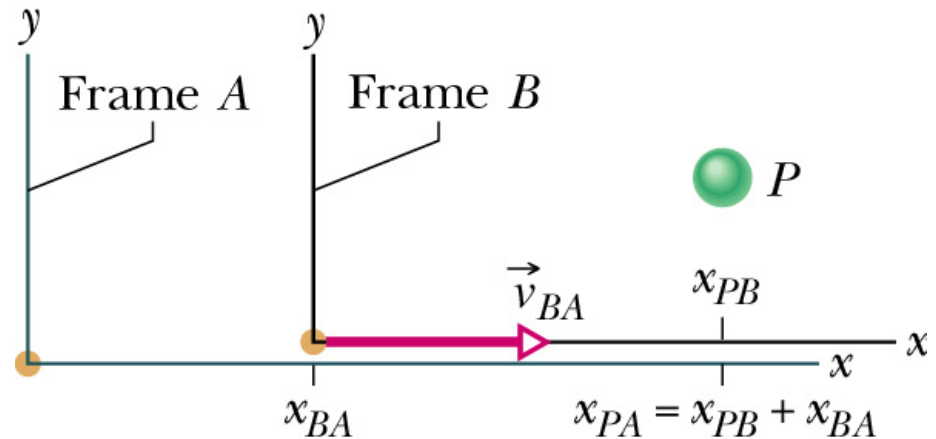


# Relative motion in one dimension

- Velocity of a particle depends on the **reference frame** of whoever is measuring the velocity

$$x_{PA} = x_{PB} + x_{BA}$$

$$v_{PA} = v_{PB} + v_{BA}$$



If the reference frame is moving at constant velocity ( $v_{BA}$  is constant):  $a_{PA} = a_{PB}$

# Careful!

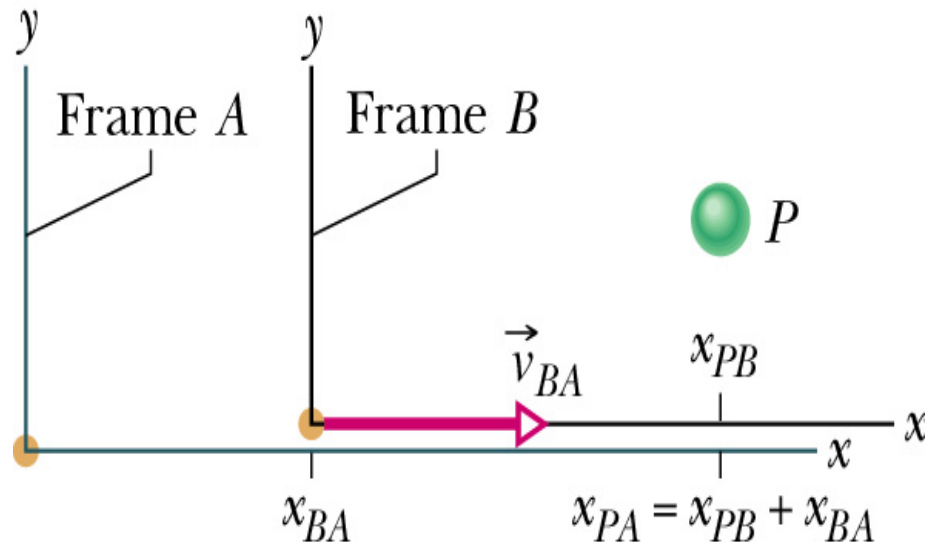


Simple addition of velocities seems logical, but we will see later (i.e., Phys2140) that this doesn't hold for very high speeds near the speed of light.

**Special Relativity!!** The failure of classical motion has to do with our fundamental assumptions of **space** and **time**.

Check point 4.7 : The table gives velocities (km/h) for Barbara and car P for three situations. For each what is the missing value and how is the distance between Barbara and car P changing.

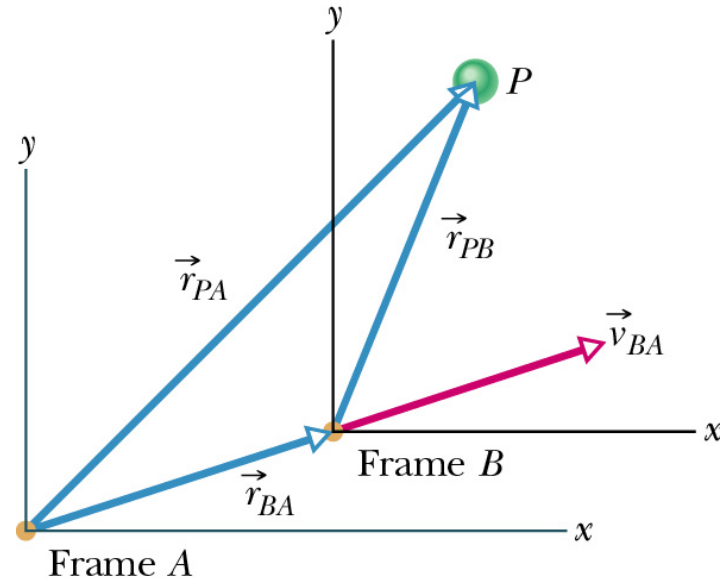
	$v_{BA}$	$v_{PA}$	$v_{PB}$
(a)	+50	+50	
(b)	+30		+40
(c)		+60	-20



# Relative motion in two dimensions

$$\mathbf{r}_{PA} = \mathbf{r}_{PB} + \mathbf{r}_{BA}$$

$$\mathbf{v}_{PA} = \mathbf{v}_{PB} + \mathbf{v}_{BA}$$



If  $\mathbf{v}_{BA}$  is constant:  $\mathbf{a}_{PA} = \mathbf{a}_{PB}$

Sample problem: A motor boat can travel at 13 km/h in still water. A river flows at 5 km/h east. A boater wishes to cross from the south bank to a point directly opposite on the north bank. At what angle must the boat be headed?