Chapter 3: Vectors

- To describe motions in 2- or 3-dimensions, we need vectors

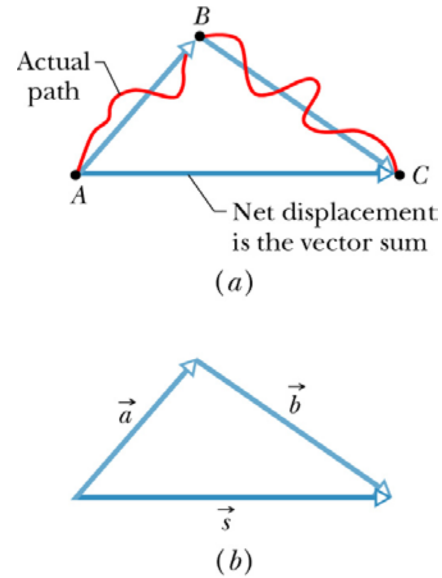
- A **vector** quantity has both a magnitude and a direction. e.g., acceleration, velocity, displacement, force, torque, and momentum.

- A **scalar** quantity does not involve a (spatial) direction. e. g. charge, mass, time, temperature, energy, etc.

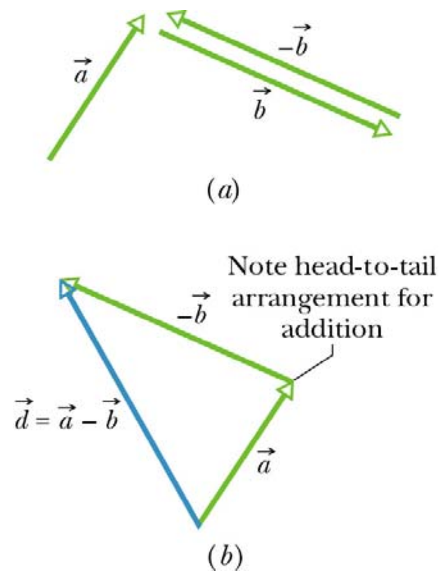
Vector addition

- Graphical method

  e. g. two vectors **a, b**



a + **b** =



a – **b** = **a** + (- **b**) =

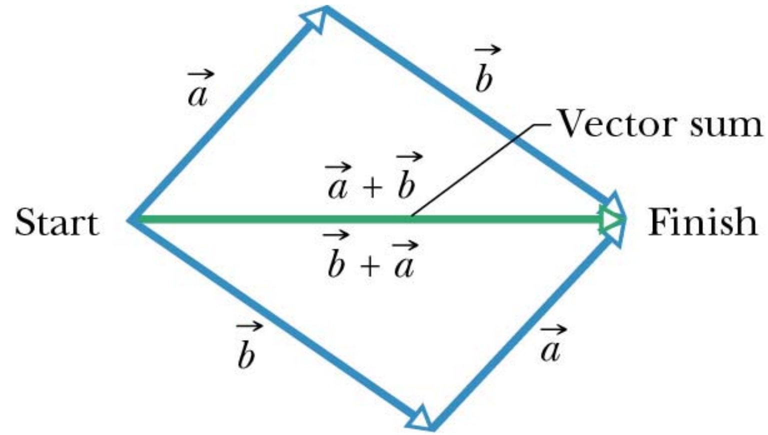
- Check point 3.1: The magnitude of displacements **a** and **b** are 3 m and 4 m, respectively, and **c** = **a** + **b**. Considering various orientations of a and b, what is

  (a) the maximum possible magnitude for **c**

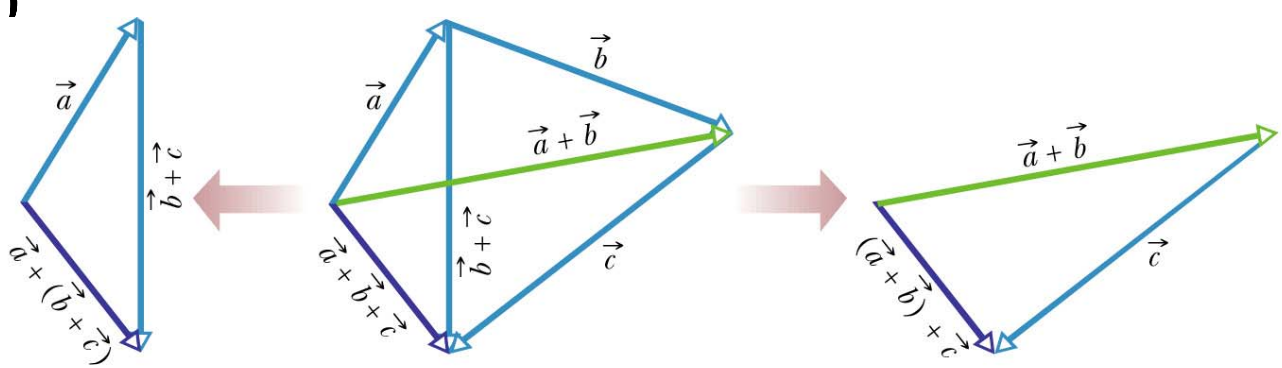
  (b) the minimum possible magnitude for **c**

Vector Addition Property

a + **b** = **b** + **a**
(commutative)

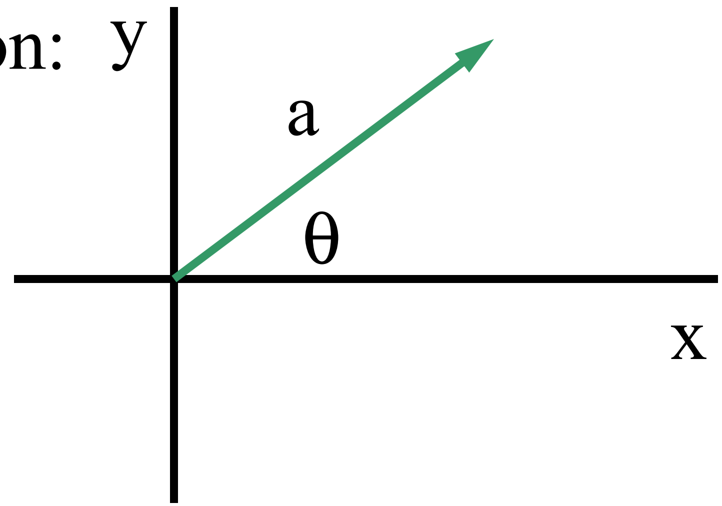


(**a** + **b**) + **c** = **a** + (**b** + **c**)
(associative)



Vector in a coordinate system

Magnitude-angle notation:



$$a_x = a\cos\theta$$

$$a_y = a\sin\theta$$

a: magnitude

θ: relative to +x direction, counter-clockwise is positive: “clock is negative”

Components of Vectors

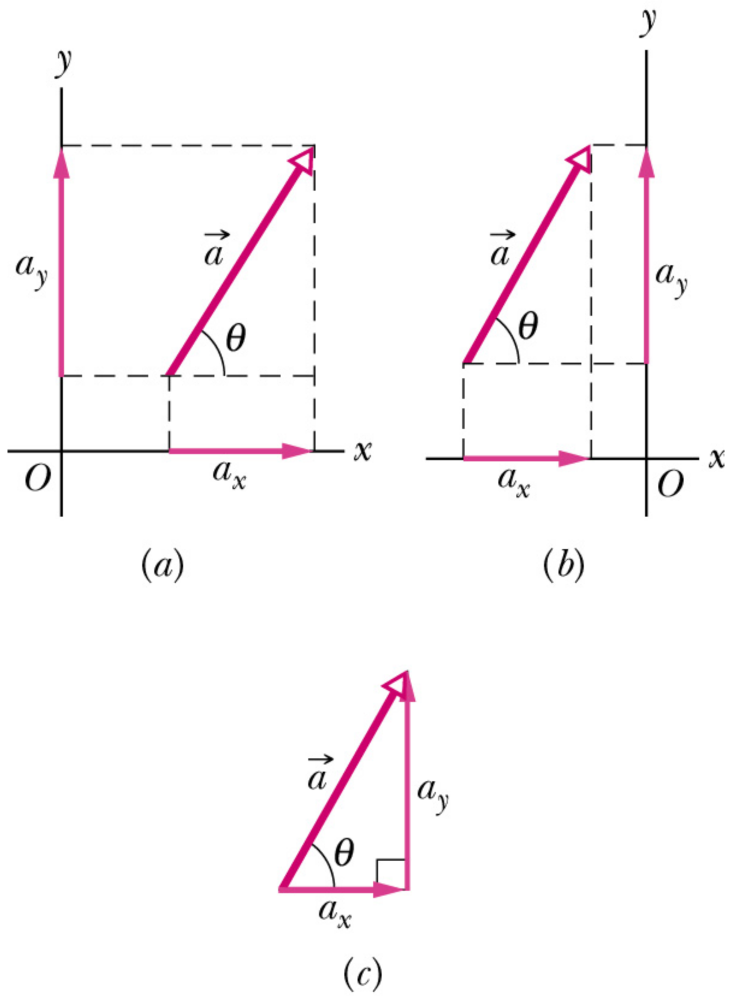
$$a_x = a\cos\theta$$

$$a_y = a\sin\theta$$

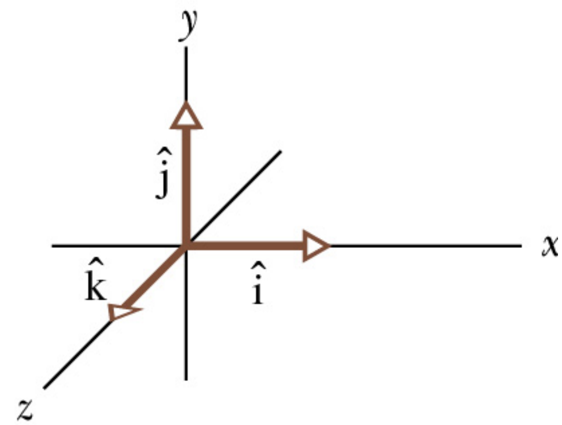
- Component notation vs magnitude-angle notation

$$a = \sqrt{a_x^2 + a_y^2}$$

$$\tan\theta = \frac{a_y}{a_x}$$



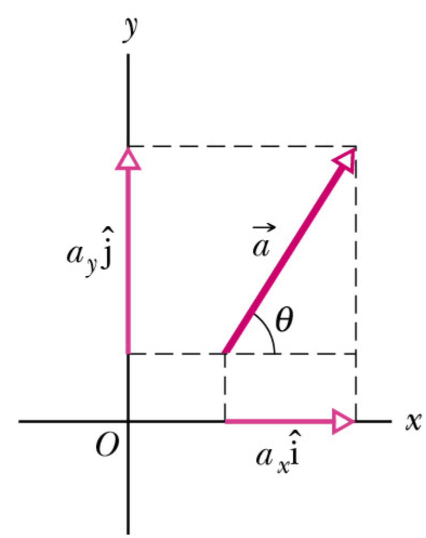
Unit Vectors



- Has a magnitude of 1 and points in a particular direction

- i, j, k, unit vectors in the positive x, y, z direction, follow right-handed coordinate system

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$



(a)

Add vectors by components

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

$$\vec{r} = \vec{a} + \vec{b} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}$$

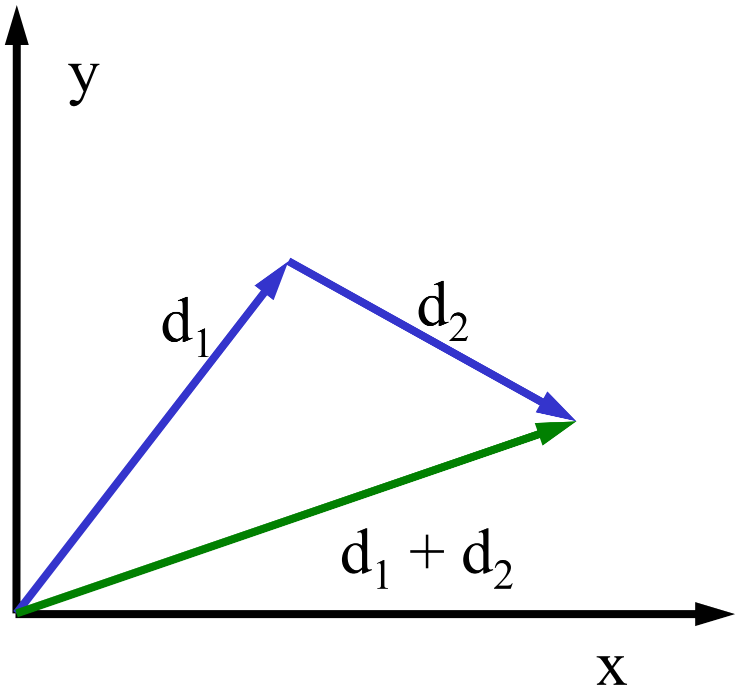
Example: In the figure below,

a) What are the signs of the x components of d_1 and d_2?

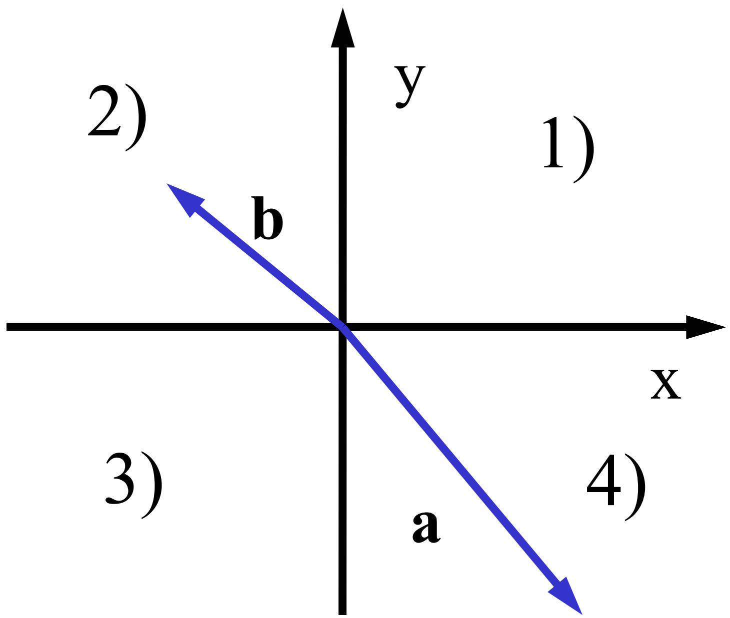
b) What are signs of the y components of d_1 and d_2?

c) What are the signs of the x and y components of $d_1 + d_2$?

d) What is the final vector $d_1 + d_2$?



A Quiz



0) none of the above

In which quadrant would **a** + **b** be located if

a = 3.0 i – 4.0 j and

b = –2.0 i + 2.0 j?

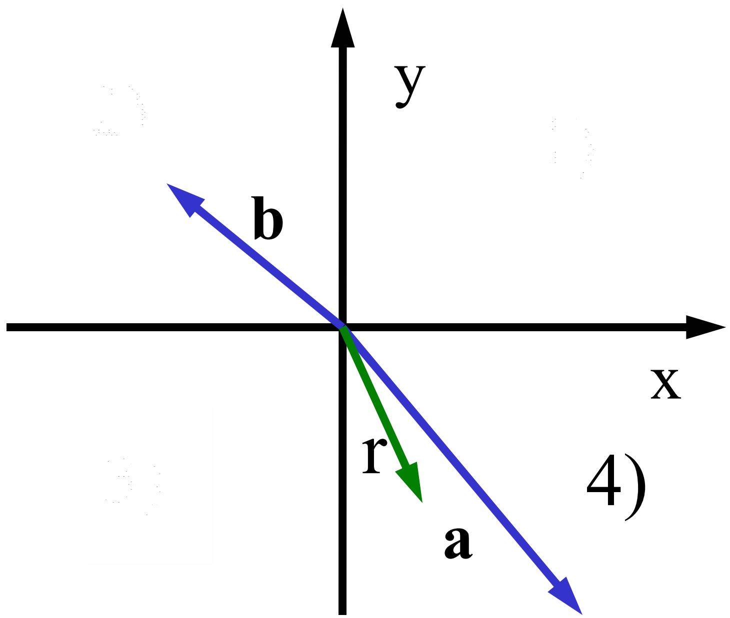
A Quiz

$$\vec{a} = 3.0\hat{i} \quad - \quad 4.0\hat{j} \quad + \quad 0.0\hat{k}$$

$$\vec{b} = -2.0\hat{i} \quad + \quad 2.0\hat{j} \quad + \quad 0.0\hat{k}$$

$$\vec{r} = \vec{a} + \vec{b} = (3.0 - 2.0)\hat{i} + (-4.0 + 2.0)\hat{j} + (0.0 + 0.0)\hat{k}$$

$$\vec{r} = (1.0)\hat{i} + (-2.0)\hat{j} + (0.0)\hat{k}$$



In which quadrant would **a** + **b** be located if

a = 3.0 i – 4.0 j and

b = –2.0 i + 2.0 j?

Multiplication of Vectors

- Multiply a vector by a scalar: **b** = s **a**
  - Magnitude of **b**: s times the magnitude of **a**
  - Direction of **b** : same as a if $s > 0$,
    opposite of a if $s < 0$
- Multiply a vector by a vector
  - Scalar product (tells you the projection of **a** onto **b**.)
    - results in a scalar
  - Vector product (tells you the area subtended by **a** and **b**.)
    - results in another vector perpendicular to both **a** and **b**.

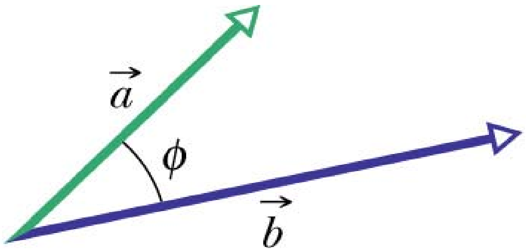
Scalar product

- Scalar product of two vectors **a** and **b**

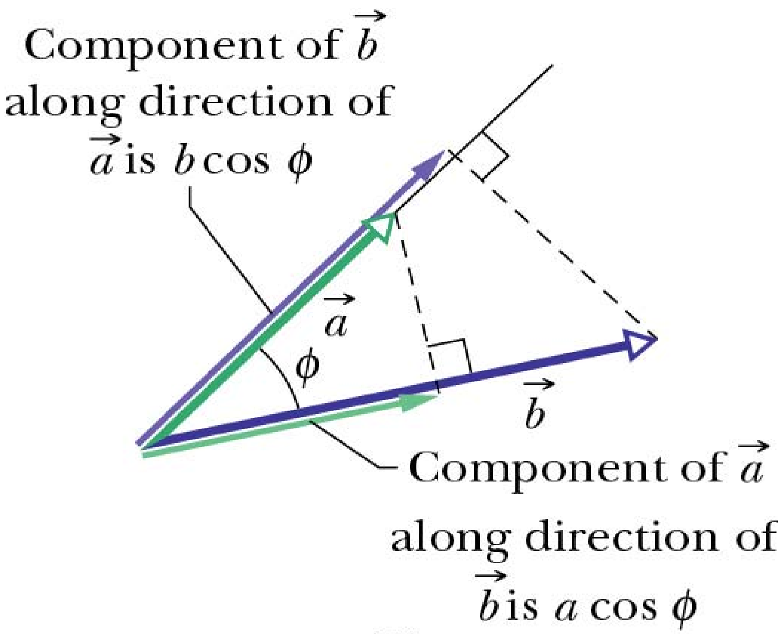
  a • b = a b cos ϕ

  a, b: magnitude of **a**, **b**

  ϕ: angle between the directions of **a** and **b**



(a)



(b)

Scalar product

$\mathbf{a}\cdot\mathbf{b} = a\ b \cos\phi$

If a and b parallel, $\phi = 0^o$

$\Rightarrow\ \mathbf{a}\cdot\mathbf{b} = a\ b\ \cos 0^o = a\ b$

if **a** and **b** perpendicular, $\phi = 90^o$

$\Rightarrow\ \mathbf{a}\cdot\mathbf{b} = a\ b \cos 90^o = 0$

$\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = 1 \qquad \mathbf{i}\cdot\mathbf{j} = \mathbf{i}\cdot\mathbf{k} = \mathbf{j}\cdot\mathbf{k} = 0$

Scalar Product

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

$$\vec{a} \bullet \vec{b} = (a_x b_x) + (a_y b_y) + (a_z b_z)$$

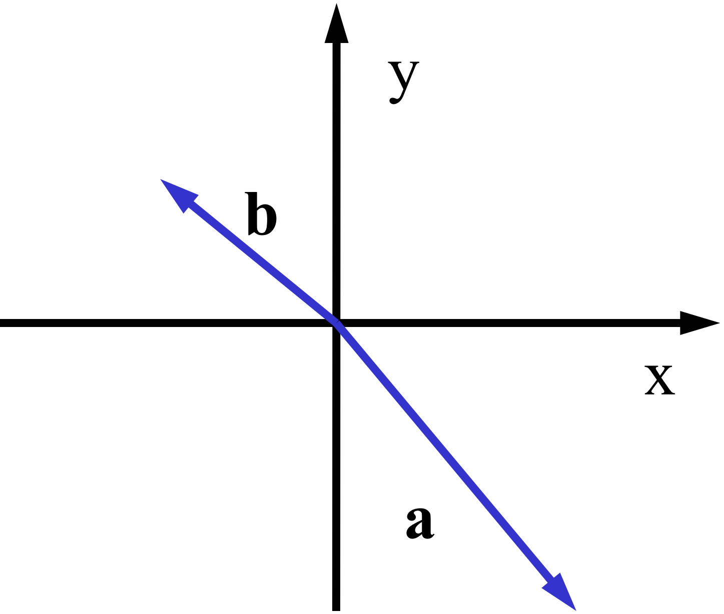
Check point 3-4: Vectors **C** and **D** have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of **C** and **D** if **C•D** is

(a) zero

(b) 12 units

(c) –12 units

Scalar Product between a and b



What is the scalar product between

a = 3.0 i – 4.0 j and

b = –2.0 i + 2.0 j?

$$\vec{a} = 3.0\hat{i} - 4.0\hat{j}$$

$$\vec{b} = -2.0\hat{i} + 2.0\hat{j}$$

$$\vec{a} \bullet \vec{b} = (3.0)(-2.0) + (-4.0)(2.0) = -14.0$$

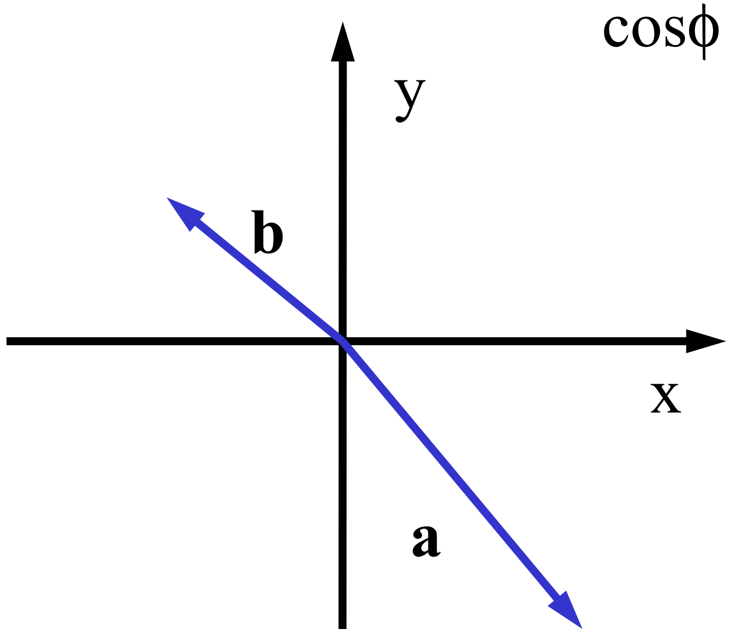
Angle between a and b

$\mathbf{a}\bullet\mathbf{b} = a\, b \cos\phi$

$a = 5$

$b = 2.828$

$\mathbf{a}\bullet\mathbf{b} = (3.0)(-2.0)+(-4.0)(2.0)+(0.0)(0.0) = -6.0 - 8.0 = -14.0$



$\cos\phi = \mathbf{a}\bullet\mathbf{b}/(a\, b) = -14.0/14.142 = -0.990$

$\phi = 171.9^o$

What is the angle between

$\mathbf{a} = 3.0\ i - 4.0\ j$ and

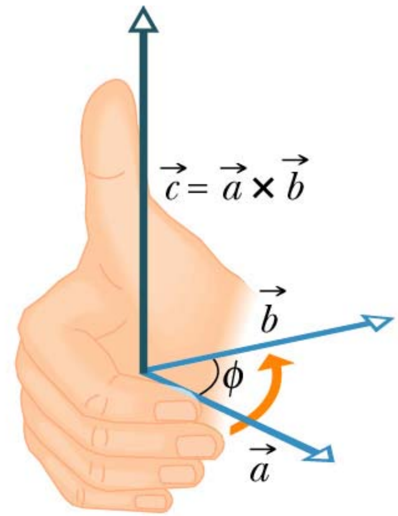
$\mathbf{b} = -2.0\ i + 2.0\ j$?

Vector Product

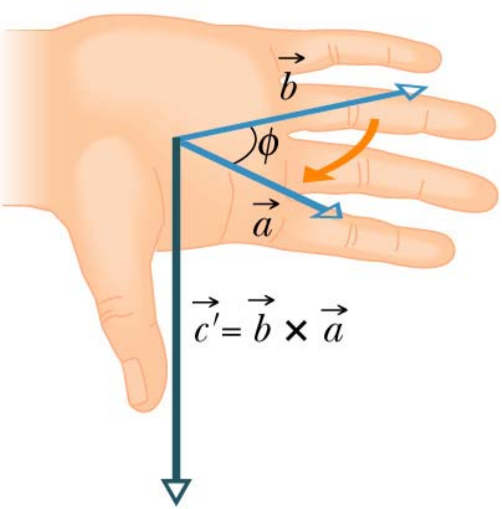
- Vector product of two vectors a and b produce a third vector c whose magnitude is

  $c = a\, b \sin \phi$

  whose direction follow the right hand rule



(*a*)



(*b*)

Note: **a** x **b** = - (**b** x **a**)

Vector product

- $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ => $c = a\, b \sin\phi$

- if $\mathbf{a}$ and $\mathbf{b}$ parallel, $\phi = 0^o$

  => $\mathbf{a} \times \mathbf{b} = 0$

  if a and b perpendicular, $\phi = 90^o$

  => $c = a\, b$

  $\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = 0$

  $\mathbf{i} \times \mathbf{j} = \mathbf{j} \times \mathbf{k} = \mathbf{k} \times \mathbf{i} = 1$

  $\mathbf{j} \times \mathbf{i} = \mathbf{k} \times \mathbf{j} = \mathbf{i} \times \mathbf{k} = -1$

Check point 3-5: Vectors **C** and **D** have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of **C** and **D** if the magnitude of the vector products **C**x**D** is

(a) Zero

(b) 12 units