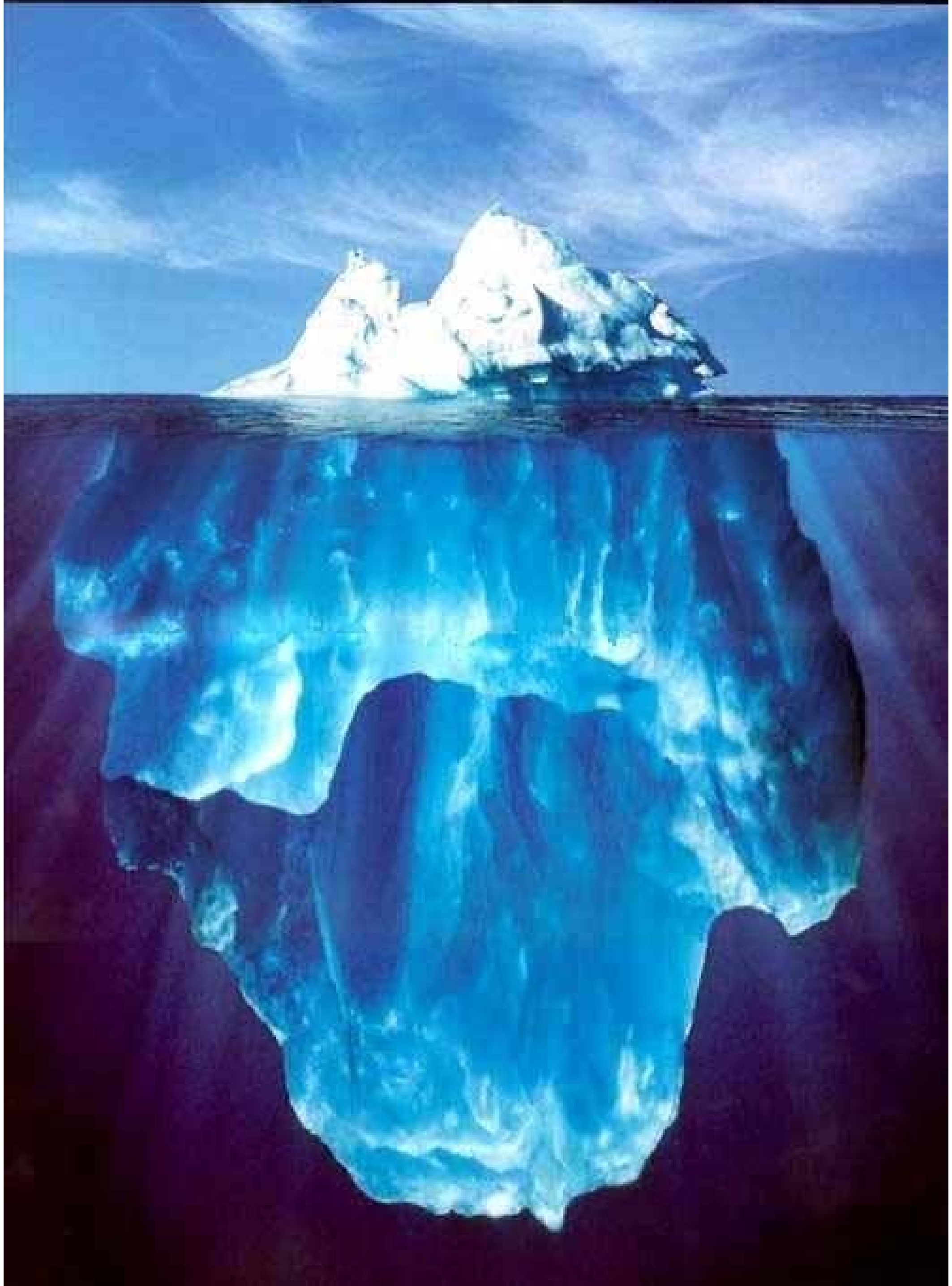


Chapter 14 Fluids

- Fluids include both liquids and gases.
- Fluid is a substance that can flow. Fluids conform to the boundaries of any container in which we put them.
- Properties of fluid: density and pressure

- **Density**: mass per unit volume for a fluid

$$\rho = \Delta m/\Delta V$$

$$\rho = m/V \qquad \text{(for uniform fluids)}$$

unit: kg/m^3

Density is a scalar.

air: 1.21 kg/m^3 (20^oC, 1atm), 60.5 kg/m^3 (20^oC, 50atm)

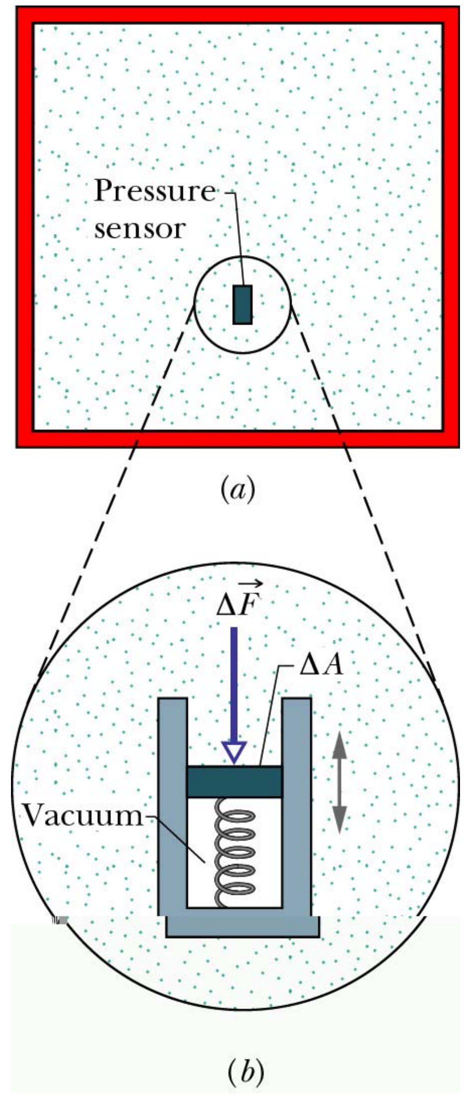
water: $1.0\text{x}10^3$ kg/m^3

- **Pressure**: force per unit area in a fluid

  $P = \Delta F / \Delta A$

  $P = F/A$ (if the force is uniform over a flat area)
- unit: 1 pascal(pa) = 1 N/m^2
- Pressure is a scalar.(same value no matter how the pressure sensor is oriented)
- Pressure of atmosphere at sea level:

  1 atm = 1.01×10^5 pa = 760 torr (mm Hg) = 14.7 lb/in^2 (psi)
- Example: estimate the atmosphere's force on the table surface



Fluids at Rest

- Hydrostatic pressure – pressure of fluids at rest

  $F_2 = F_1 + mg$

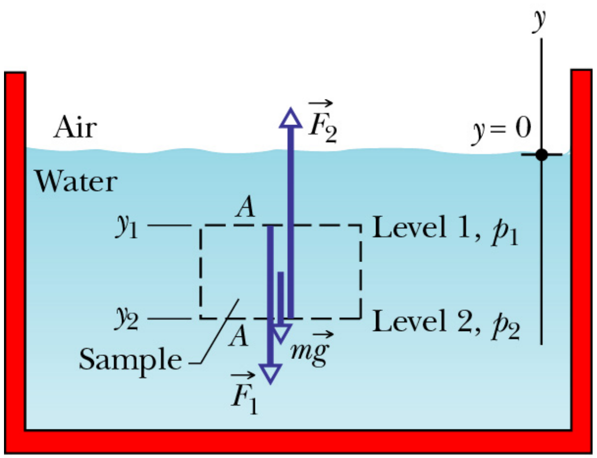
  since $F = PA$, $m = \rho V = \rho A(y_1\text{-}y_2)$

  $p_2A = P_1A + \rho Ag(y_1\text{-}y_2)$

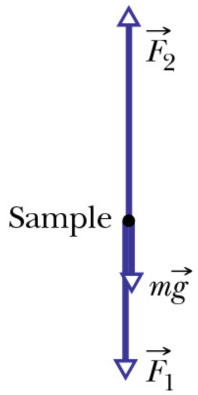
  yields:

  $\mathbf{p_2 = p_1 + \rho\, g\, (\, y_1 - y_2\,) = p_1 + \rho\, g\, h}$

  The deeper in the liquid, the higher the pressure, does not depend on any horizontal dimension of the fluid or its container.



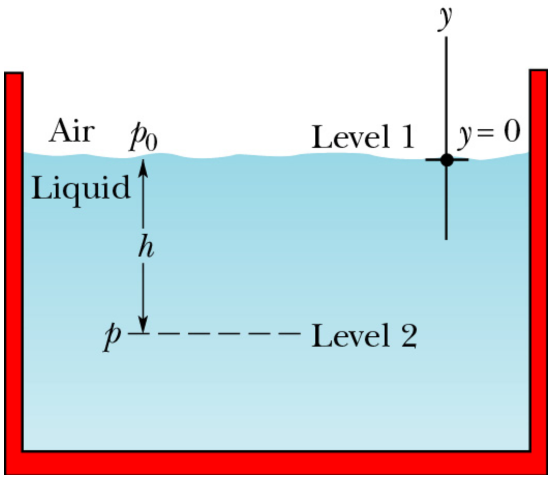
(a)



(b)

- **Pressure of fluid: $p_2 = p_1 + \rho g h$**
  - **Level 1 is above level 2**

- Pressure at depth *h* in the liquid:



$p_1 = p_0$, $h = h$, $p_2 = p$

then $\quad p = p_0 + \rho g h$

- Atmospheric pressure of a distance *d* above sea level

$p_1 = p$, $h = d$, $p_2 = p_0$

then $\quad p_0 = p + \rho g d$

$$p = p_0 - \rho g d$$

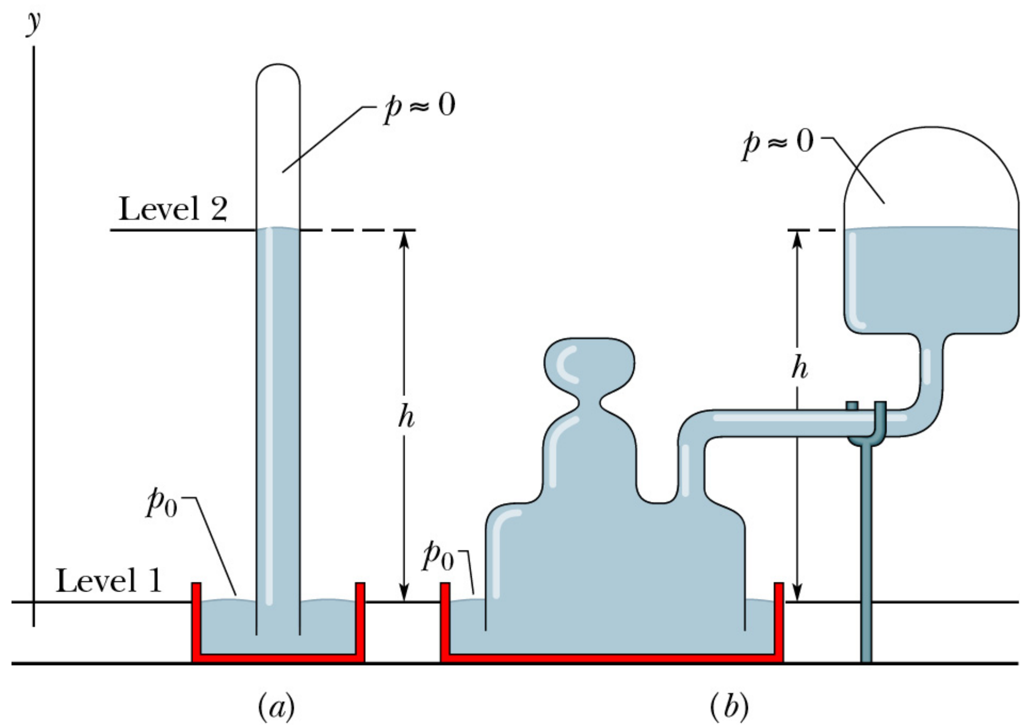
Measuring Pressure

- The mercury barometer -- a device to measure the pressure of the atmosphere

  $h = h$, $p_1 = 0$, $p_2 = p_0$

  then $p_0 = 0 + \rho g h$

  $p_0 = \rho g h$

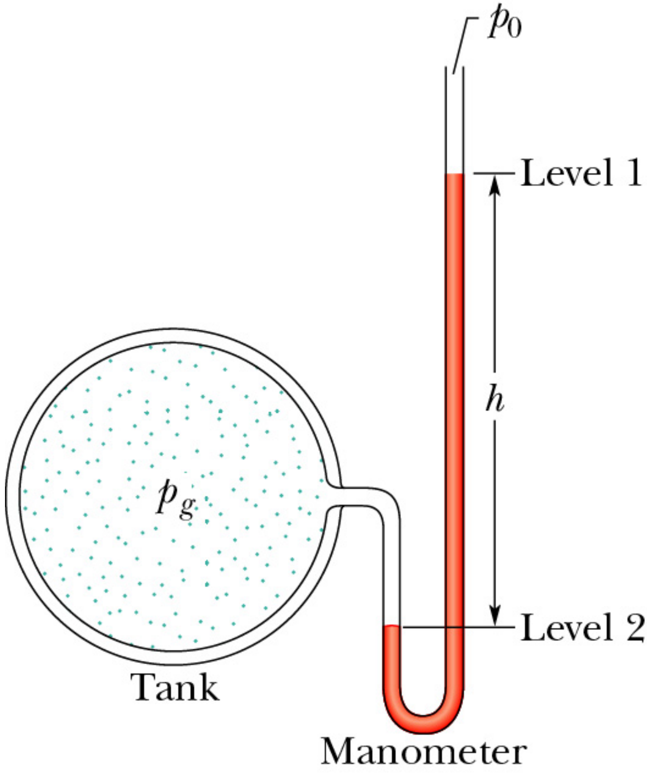


- Absolute pressure = total pressure
- Gauge pressure = absolute pressure – atmospheric pressure
  – e.g. The pressure meter for the car tire
- The open-tube manometer – a device to measure the gauge pressure of a gas

$p_1 = p_0, \quad p_2 = p_a, \quad h = h$

then $\quad p_a = p_0 + \rho\, g\, h$

$p_g = p_a - p_0 = \rho\, g\, h$



Pascal’s Principle

Pascal’s principle: a change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

e. g. squeeze toothpaste, balloon

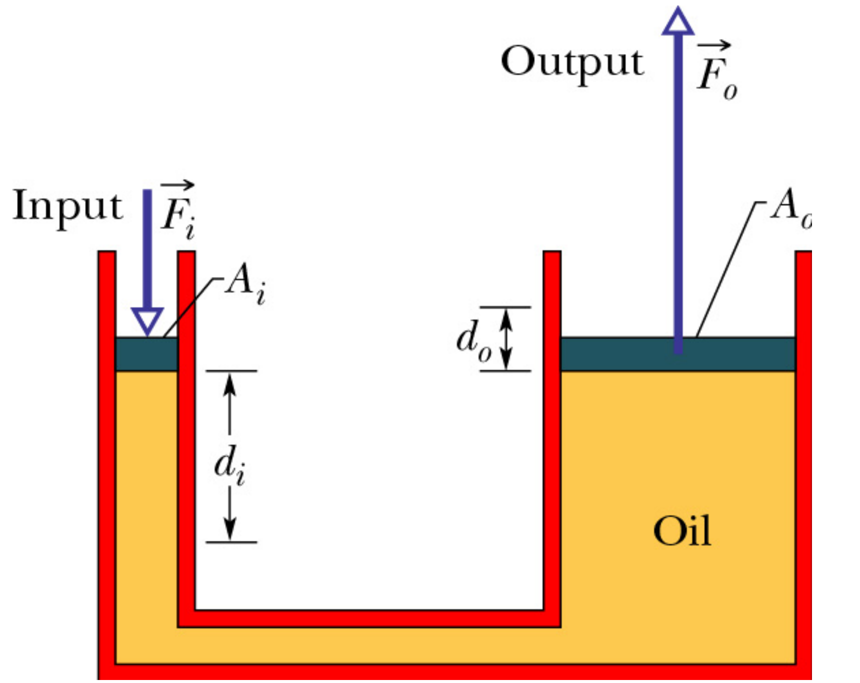
Pascal's Principle and the Hydraulic Level

$\Delta p_i = \Delta p_o \quad F_i/A_i = F_o/A_o$

$F_o = F_i\,(A_o/A_i)$

If $A_o = 100A_i$, $F_o = 100F_i$

Small force lifts heavy things…



$V = A_i d_i = A_o d_o$

$d_o = d_i\,(A_i/A_o)$

If $A_o = 100A_i$, $d_i = 100d_o$

…. by moving longer distance.

$W_o = F_o d_o = (F_i A_o/A_i)\,(d_i A_i/A_o) = F_i d_i = W_i$

Work input equals work output.

Achimedes' Principle

- When a body is fully or partially submerged in a fluid, a **buoyant force** F_b from the surrounding fluid acts on the body.

  direction: upward

  magnitude: the weight of the fluid that has been displaced by the body.

$$F_b = m_f g = \rho_f V_f g$$

  - stone in water
  - wooden block in water

For a floating object

$$F_g = F_b = \rho_f V_f g$$

the magnitudes of the gravity and the buoyant force are equal.

The object will sink until it displaces equal mass of liquid as its mass

However,

If the object has a greater density than the liquid, then it will sink.

Apparent weight in a fluid $W_{apparent} = W_{actual} - F_b$

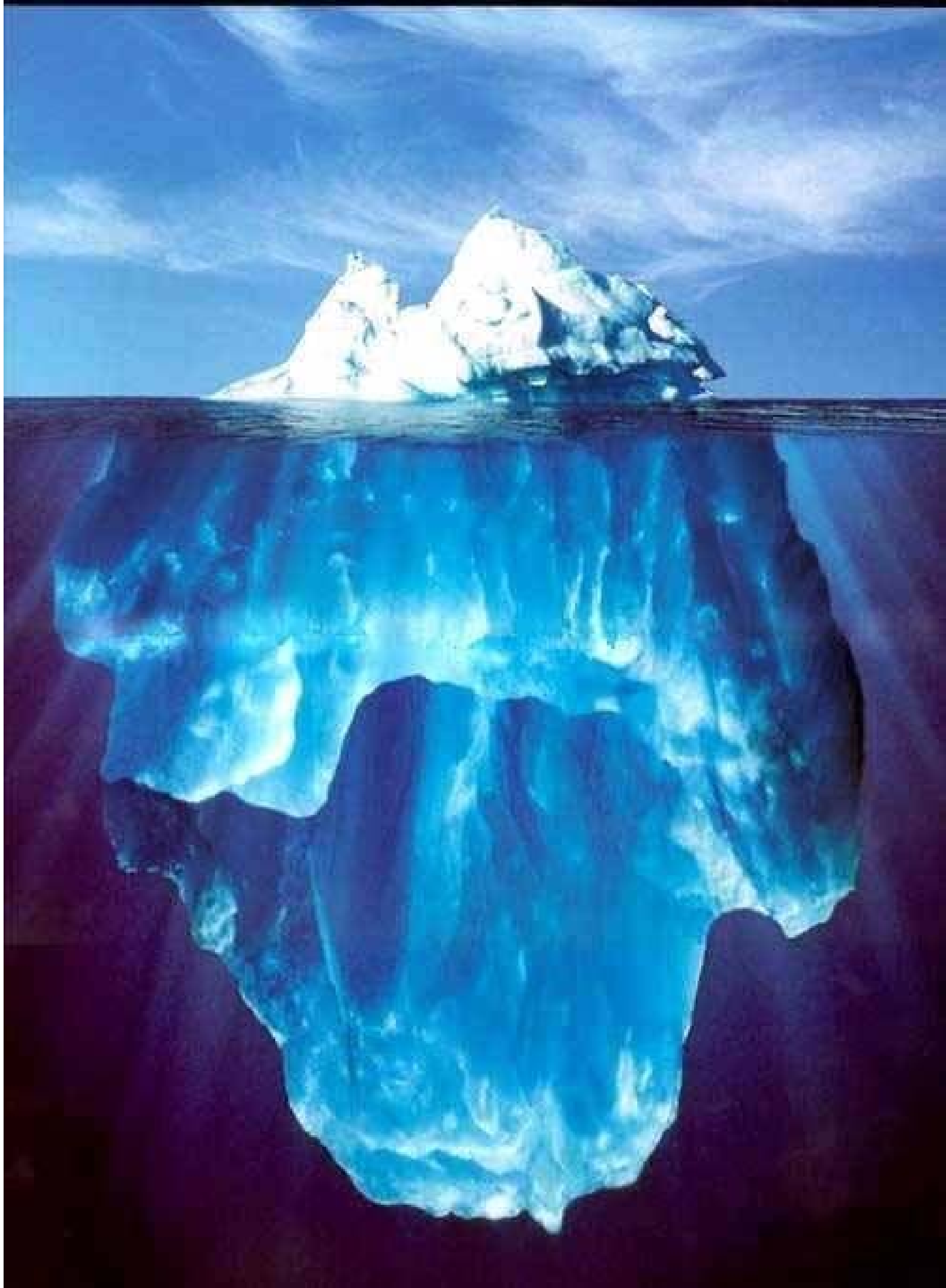
Sample Problem 14-4
What fraction of the iceberg is above water?

V_i = total volume of iceberg
V_f = volume of displaced water

$$\text{frac} = \frac{V_i - V_f}{V_i} = 1 - \frac{V_f}{V_i}$$

Key point: Forces balance since the iceberg is at rest.

$m = \rho V$ (density x volume)
$\Rightarrow mg = \rho V g$
$m_i\, g = \rho_i V_i\, g = m_f\, g = \rho_f V_f\, g$



Sample Problem 14-4
What fraction of the iceberg is above water?

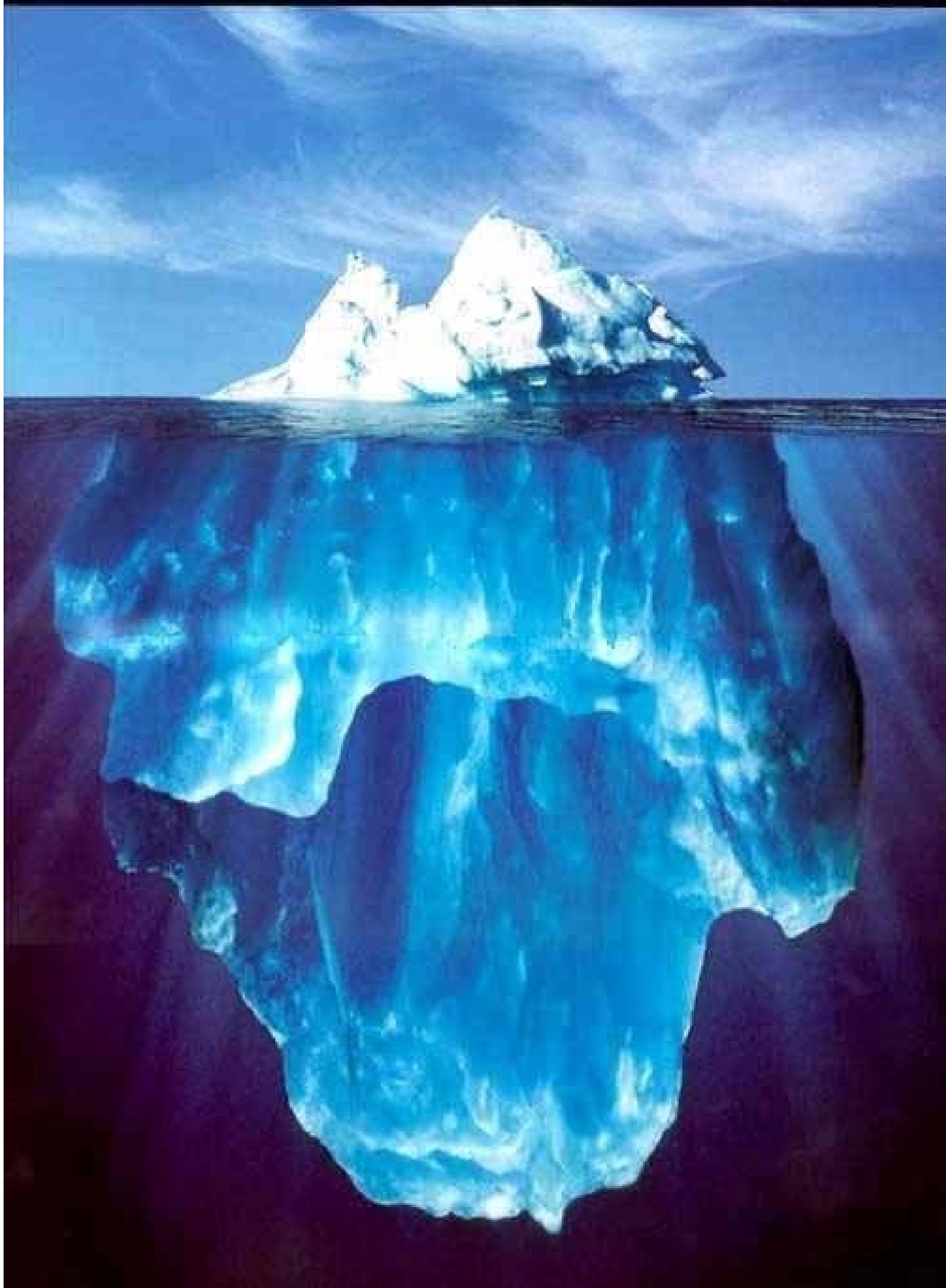
V_i = total volume of iceberg
V_f = volume of displaced water

Key point: Forces balance since the iceberg is at rest.

$$m_i = \rho_i V_i = m_f = \rho_f V_f$$
$$\rho_i V_i = \rho_f V_f$$

$$\text{frac} = 1 - \frac{V_f}{V_i} = 1 - \frac{\rho_i}{\rho_f}$$

$$= 1 - \frac{917\,\text{kg/m}^3}{1024\,\text{kg/m}^3} = 0.10$$

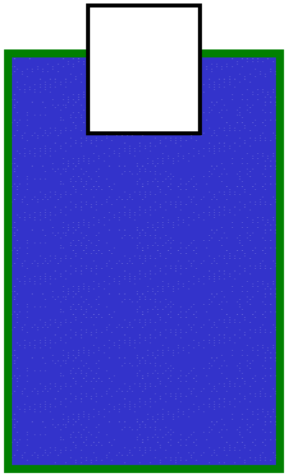


A Quiz

$\rho_{water} = 1000\ kg/m^3$ $\rho_{ice} = 917\ kg/m^3$

A block of ice at 0^oC is floating on the surface of ice water in a beaker. The surface of the water just comes to the top of the beaker. When the ice melts the water level will:

1) Depends on the initial ratio of water to ice
2) Fall
3) Rise and overflow will occur
4) Depends on the shape of the block of ice
5) Remains the same



A Quiz

$\rho_{water} = 1000\ kg/m^3$ $\rho_{ice} = 917\ kg/m^3$

Ice displaces equal weight of water => floating part “fills in” the submersed part as it melts.

A block of ice at 0^oC is floating on the surface of ice water in a beaker. The surface of the water just comes to the top of the beaker. When the ice melts the water level will:

1) Depends on the initial ratio of water to ice
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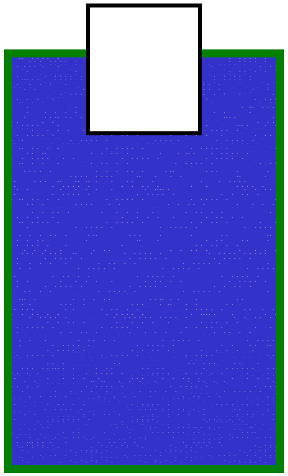
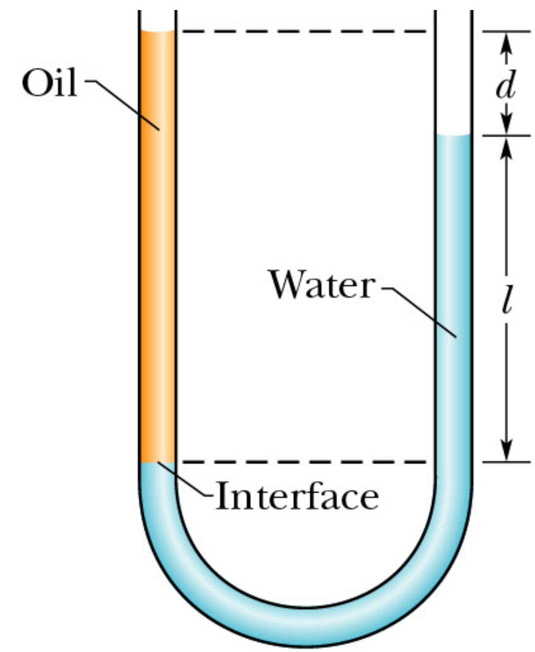
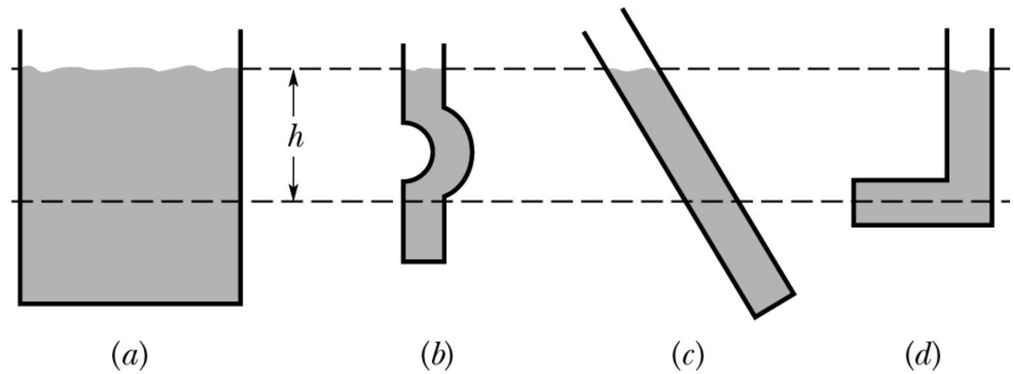


- Sample problem 14-3. The U – tube contains two liquids in static equilibrium. Water of density $\rho_w = 998$ k/m^3 is in the right arm, and oil of unknown density ρ_x is in the left. $l = 135$ mm, $d = 12.3$ mm. What is the density of the oil?



- Check point 14-1: The figure shows four containers of olive oil. Rank them according to the pressure at depth h, greatest first.



Ideal fluid in motion

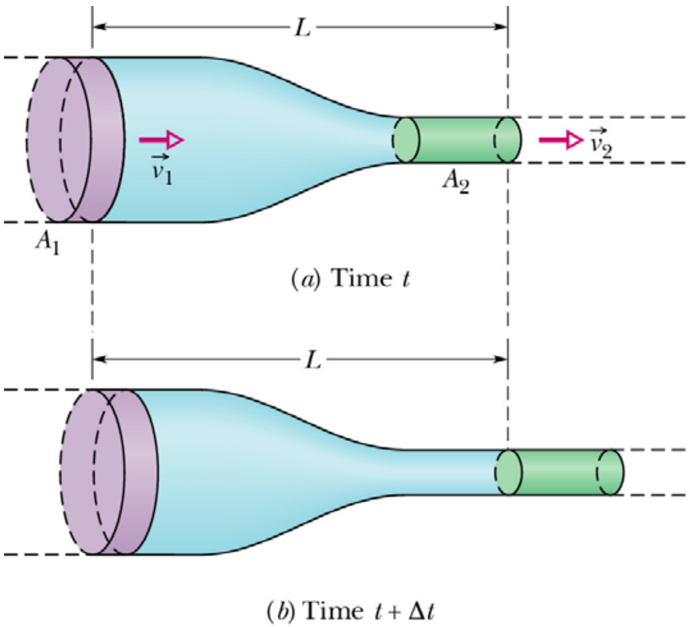
- Steady flow

- Incompressible flow
- Nonviscous flow
- Irrotational flow

Mechanical Energy will be conserved under these restrictions.

- Next, we will discuss two principles related to ideal fluid in motion.

The equation of continuity



$\Delta V = A\ \Delta x = A\ v\ \Delta t$

$\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t$

$A_1 v_1 = A_2 v_2$ (equation of continuity)

thus Av = constant (volume flow rate)

– a nozzle or your thumb over a garden hose

Bernoulli's equation

Bernoulli's equation is based on
Conservation of Mechanical Energy

$$\mathbf{p + \tfrac{1}{2}\,\rho v^2 + \rho g y = constant}$$

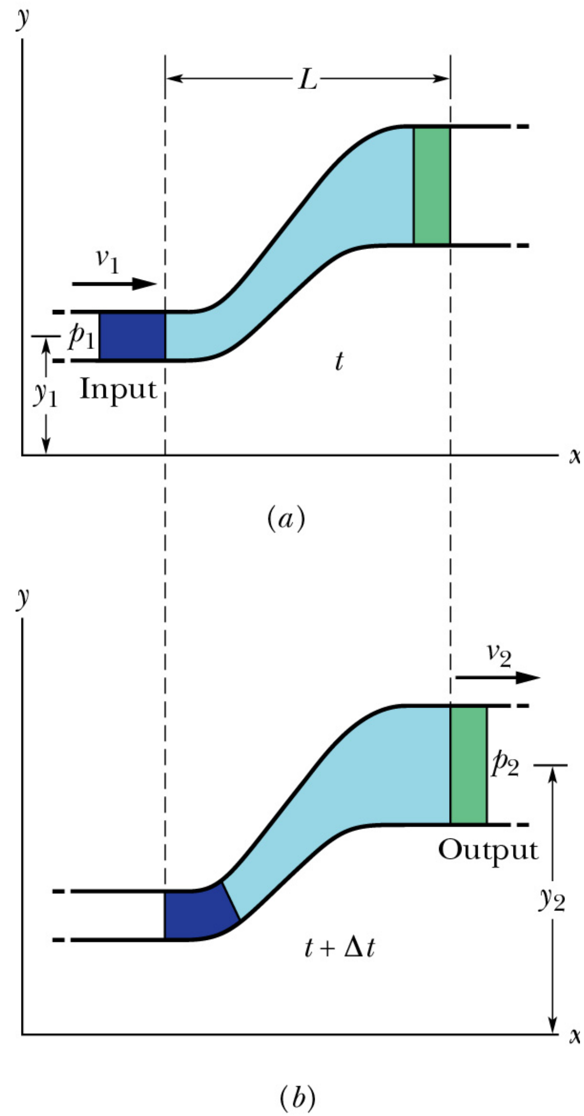
Pressure

Kinetic energy per volume

Potential energy per volume

=

Conserved



$$\mathbf{p_1 + \tfrac{1}{2}\,\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\,\rho v_2^2 + \rho g y_2 = constant}$$

Bernoulli's equation

Bernoulli's equation

based on Conservation of Mechanical Energy

$$\mathbf{p + \tfrac{1}{2}\rho v^2 + \rho g y = constant}$$

For fluid at rest, v = 0

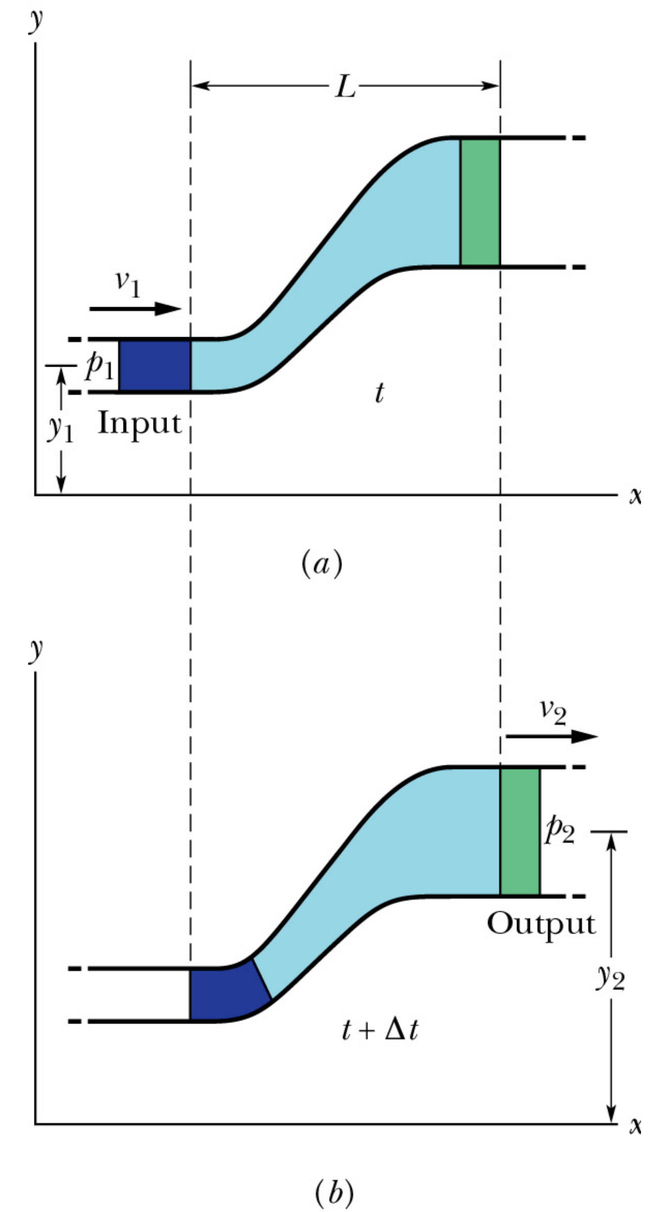
$$p + \rho g y = \text{constant}$$

$$p_2 = p_1 + \rho g(y_1 - y_2)$$

For y = constant,

$$p + \tfrac{1}{2}\rho v^2 = \text{constant}$$

if v increases, then p decreases



Bernoulli's equation

Bernoulli's equation

based on Conservation of Mechanical Energy

$$\mathbf{p + \tfrac{1}{2}\,\rho v^2 + \rho g y = constant}$$

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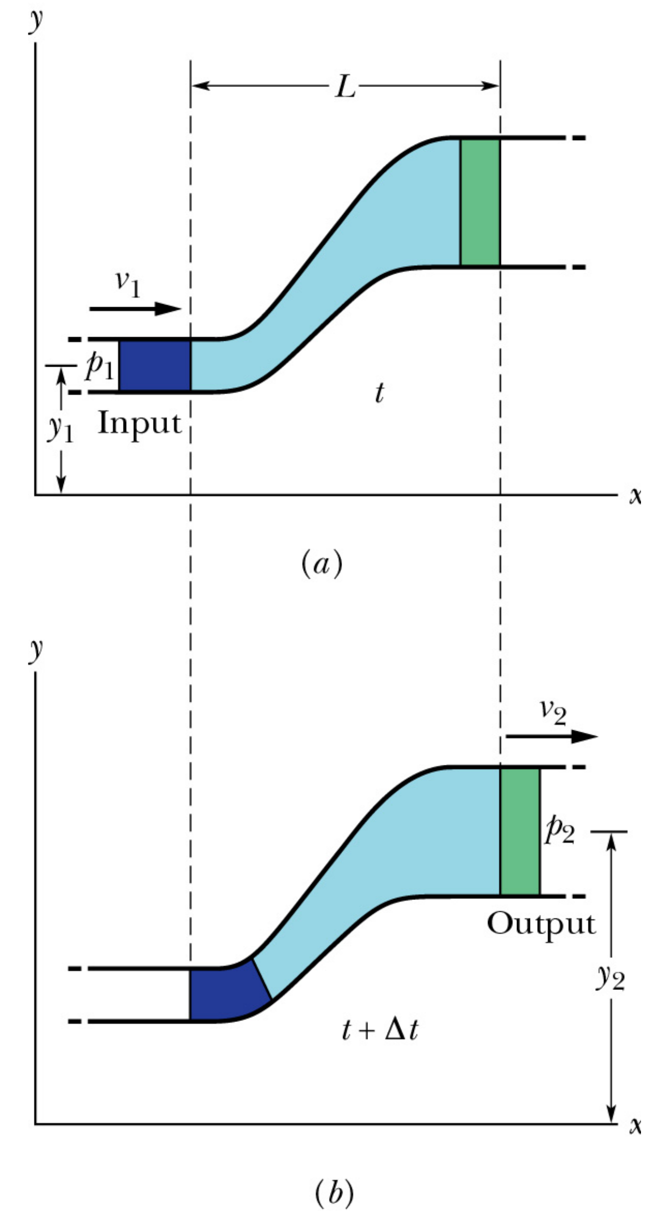

Bernoulli's equation

Bernoulli's equation

based on Conservation of Mechanical Energy

$$\mathbf{p + \tfrac{1}{2} \rho v^2 + \rho g y = constant}$$

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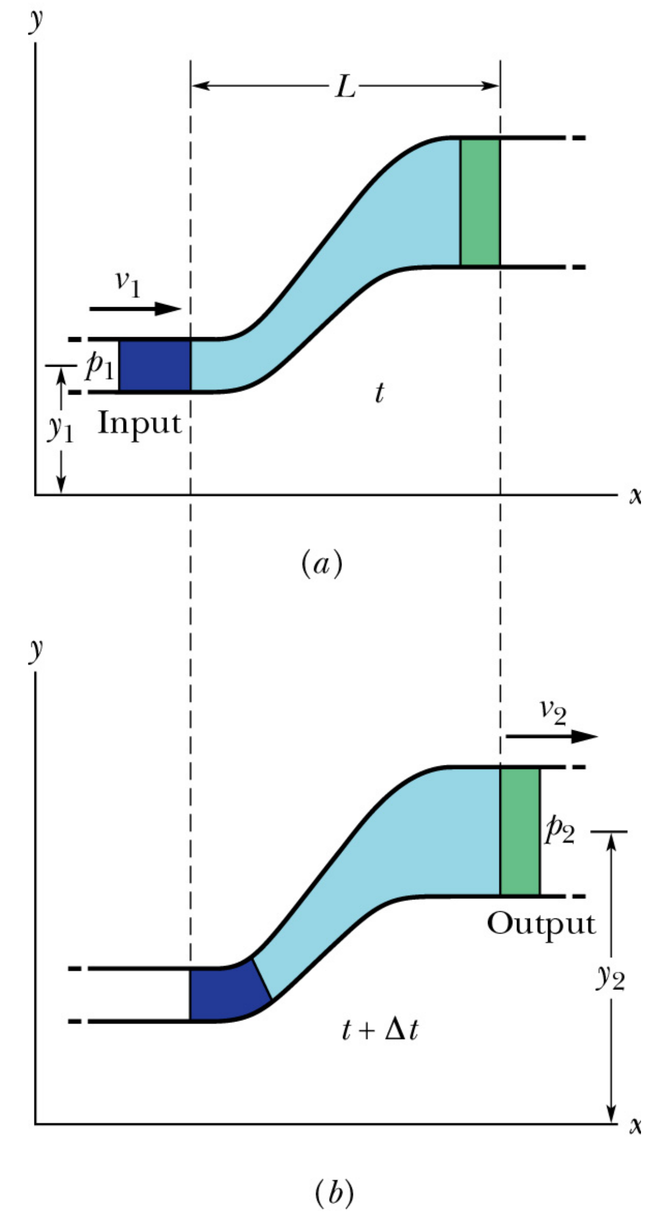


Airfoils

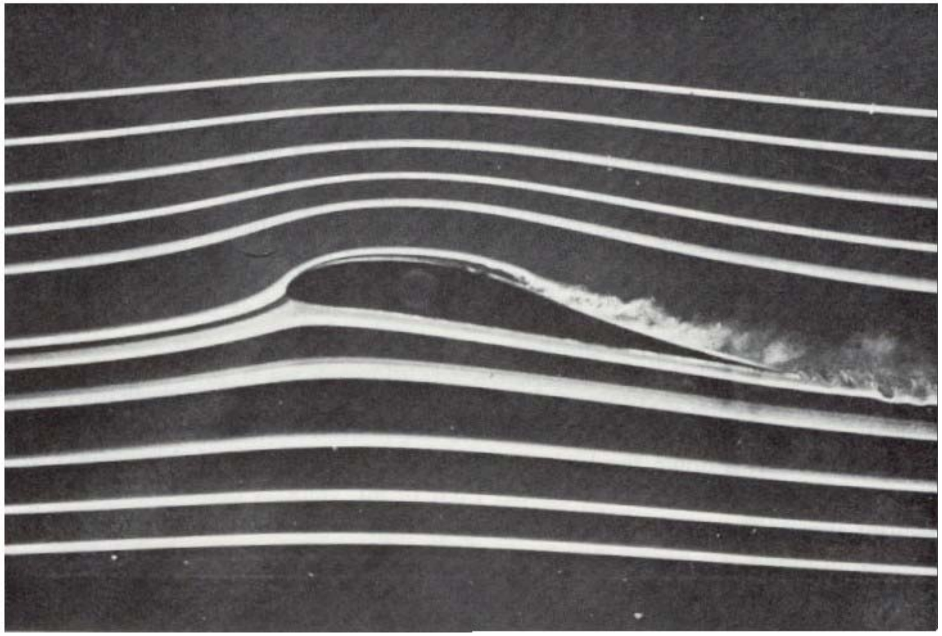


Figure 2.4 Smoke photograph of the low-speed flow over a Lissaman 7769 airfoil at 10° angle of attack. The Reynolds number based on chord is 150,000. This is the airfoil used on the Gossamer Condor man-powered aircraft. *(The photograph was taken in one of the Notre Dame University smoke tunnels by Dr. T. J. Mueller, Professor of Aerospace Engineering at Notre Dame, and is shown here through his courtesy.)*

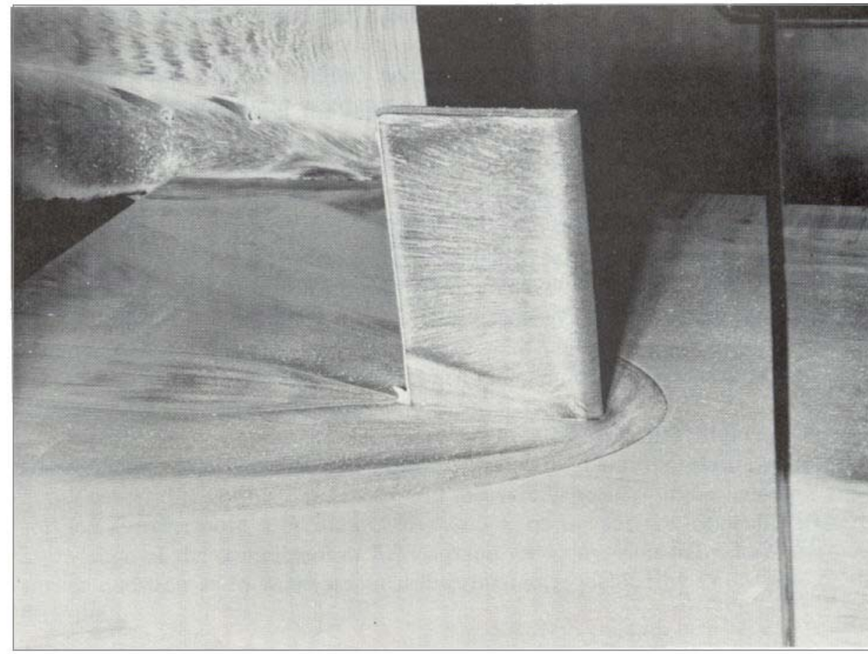
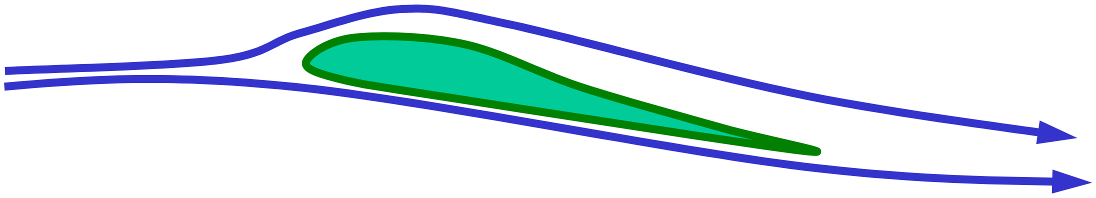


Figure 2.5 An oil streak photograph showing the surface streamline pattern for a fin mounted on a flat plate in supersonic flow. The parabolic curve in front of the fin is due to the bow shock wave and flow separation ahead of the fin. Note how clearly the streamlines can be seen in this complex flow pattern. Flow is from right to left. The Mach number is 5 and the Reynolds number is 6.7×10^6. *(Courtesy of Allen E. Winkelmann, University of Maryland, and the Naval Surface Weapons Center.)*

from Anderson, *Introduction to Flight*, 3rd ed.1978

Airfoils

top airstream moves faster than the lower airstream.



Consider

$v_{lower} = 70\text{m/s}$ ← (160 mph -- takeoff speed)

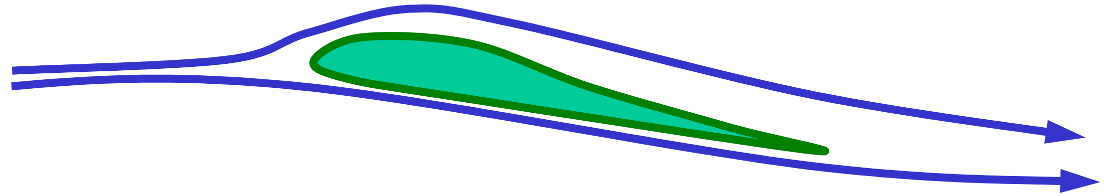
$p_{lower} = p_{atm}$

$\rho_{air} = 1.21\text{kg/m}^3$

The shape of the airfoil causes $v_{upper} = 120\text{m/s}$.

Find $\Delta p_{upper} = p_{lower} - p_{upper}$

Airfoils



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$\rho_{air} = 1.21\text{kg/m}^3$

The shape of the airfoil causes $v_{upper} = 120\text{m/s}$.

Find $\Delta p_{upper} = p_{lower} - p_{upper}$

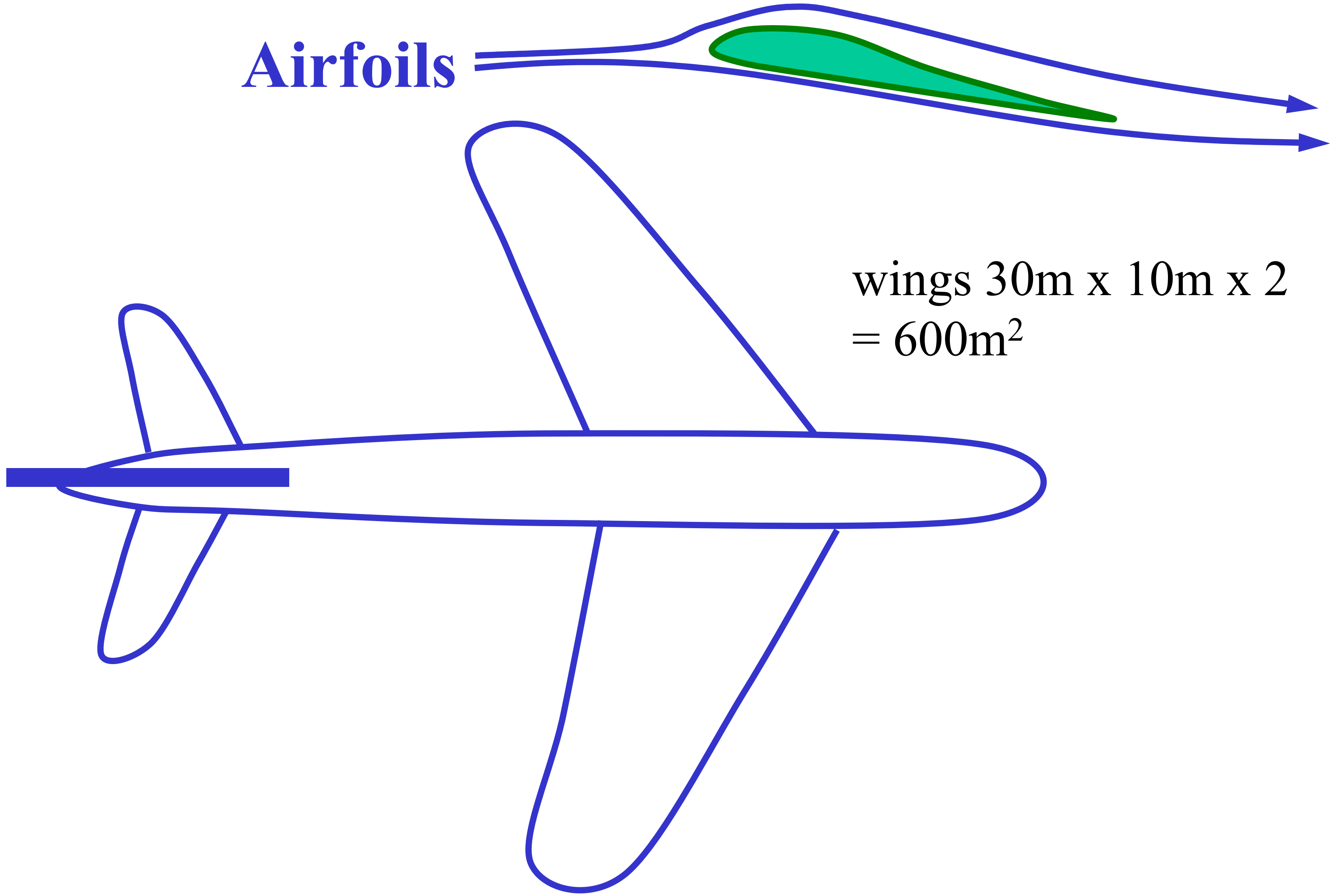
$$p_1 + \tfrac{1}{2}\rho v_1^2 + \cancel{\rho g y_1} = p_2 + \tfrac{1}{2}\rho v_2^2 + \cancel{\rho g y_2} = \text{constant}$$

$$p_{upper} + \tfrac{1}{2}\rho v_{upper}^2 = p_{lower} + \tfrac{1}{2}\rho v_{lower}^2$$

$$\Delta p = p_{lower} - p_{upper} = \frac{1}{2}\rho_{air} v_{upper}^2 - \frac{1}{2}\rho_{air} v_{lower}^2$$

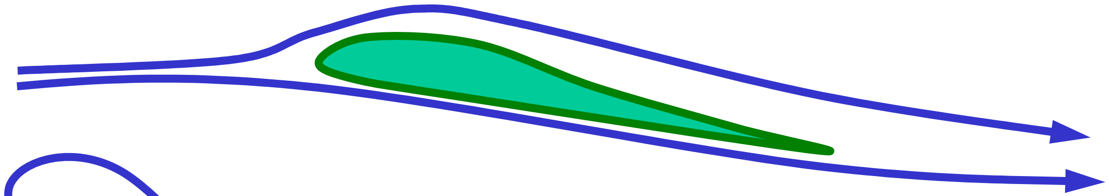
$$= \frac{1}{2}\rho_{air}\left(v_{upper}^2 - v_{lower}^2\right) = \left(\frac{1}{2}\right)(1.21)\left(120^2 - 70^2\right) = 5{,}748\text{N/m}^2$$

Airfoils



wings 30m x 10m x 2
= $600m^2$

Airfoils



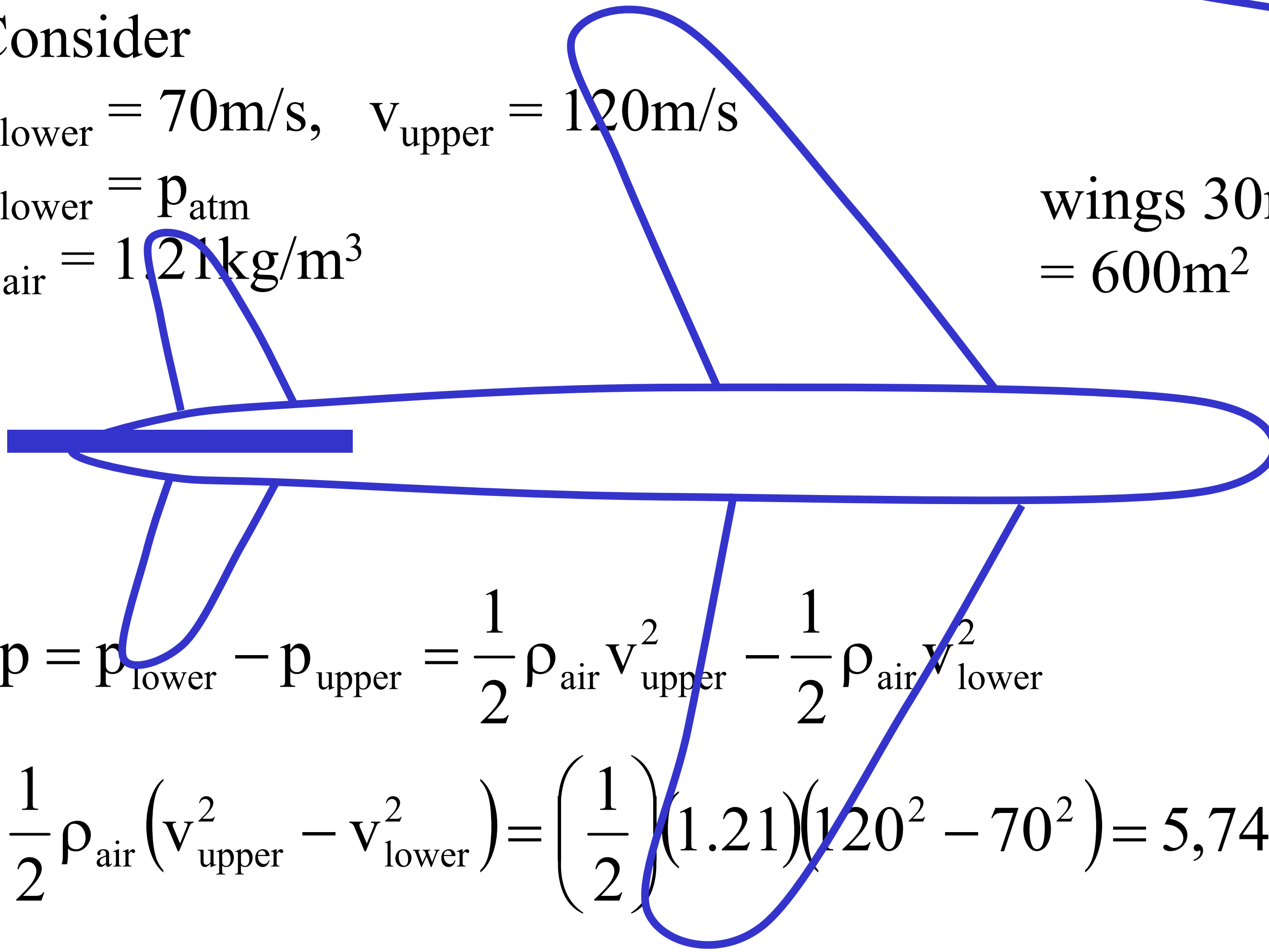
Consider

$v_{lower} = 70\text{m/s}, \quad v_{upper} = 120\text{m/s}$

$p_{lower} = p_{atm}$

$\rho_{air} = 1.21\text{kg/m}^3$

wings 30m x 10m x 2
$= 600\text{m}^2$



$$\Delta p = p_{lower} - p_{upper} = \frac{1}{2}\rho_{air} v_{upper}^2 - \frac{1}{2}\rho_{air} v_{lower}^2$$

$$= \frac{1}{2}\rho_{air}\left(v_{upper}^2 - v_{lower}^2\right) = \left(\frac{1}{2}\right)(1.21)\left(120^2 - 70^2\right) = 5{,}748\text{N/m}^2$$

=> 5,748 x 600 = 3,450,000 N of lift!!! **(=776,250 lbs = 388 tons)**