Chapter 12 Equilibrium & Elasticity

If there is a net force, an object will experience a linear acceleration. (period, end of story!)

If there is a net torque, an object will experience an angular acceleration. (period, end of story!)

How can we keep things from moving, then?

Recall, $\vec{F}_{net} = \dfrac{d\vec{P}}{dt}$ and $\vec{\tau} = \dfrac{d\vec{L}}{dt}$

Chapter 12 Equilibrium & Elasticity

- Equilibrium: $\vec{P}$ = constant and $\vec{L}$ = constant

- Static equilibrium: Objects that are not moving either in translation or rotation. $\vec{P} = 0$ $\vec{L} = 0$

- Requirements of equilibrium

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} \qquad \vec{F}_{net} = 0 \quad \text{(balance of forces)}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} \qquad \tau = 0 \quad \text{(balance of torques)}$$

- For the situations in which the forces that act on the body lie in the xy plane. Then the only torque that can act on the body is τ_z . We have

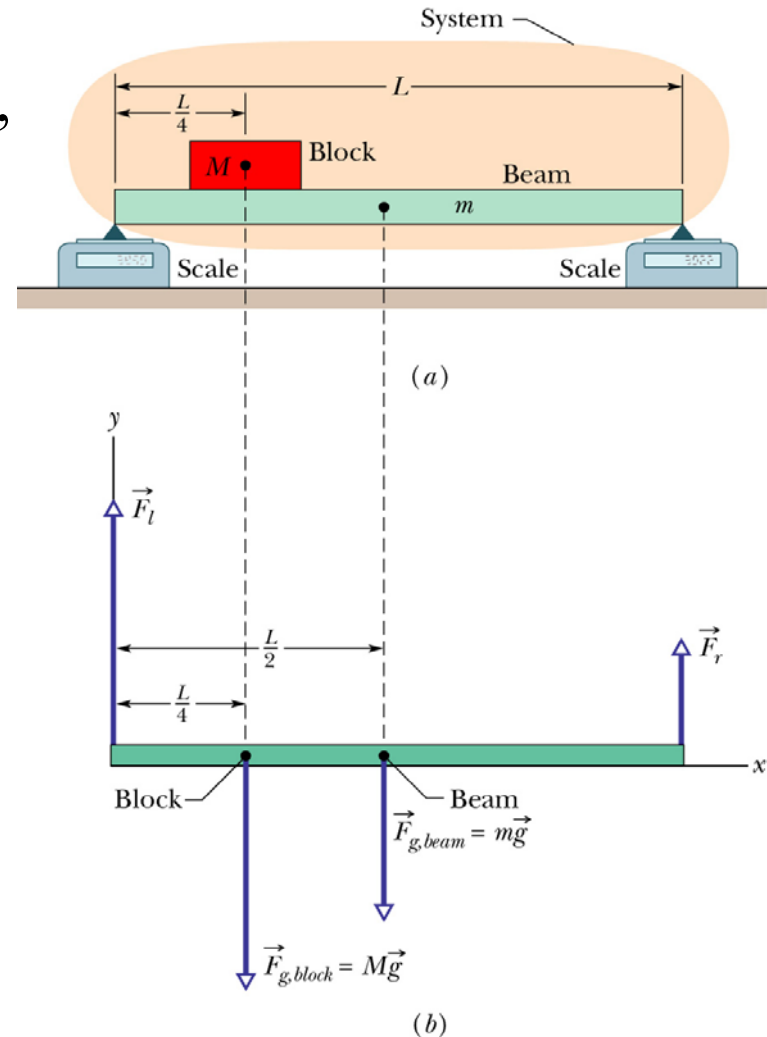$$F_{net,x} = 0 \qquad F_{net,y} = 0 \qquad \tau_{net,z} = 0$$

  - $\tau_{net,z}$ is the net torque that the external forces produce either about the z axis or about any axis parallel to it

Center of gravity

- Center of gravity: gravitational force on a body effectively act at a single point.

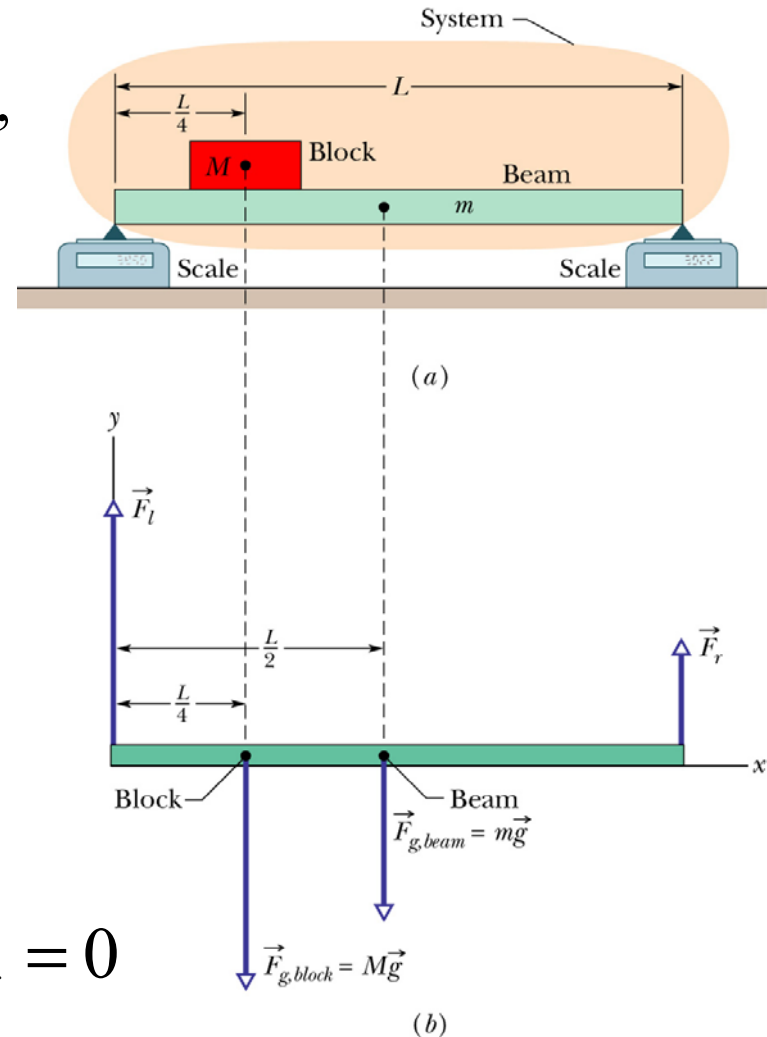- For everyday objects, the center of gravity coincident with its center of mass.

Sample Problem 12-1

A beam of length L and mass m = 1.8 kg, is at rest with its ends on two scales. A block of M = 2.7 kg, is at rest on the beam, with its center a distance L/4 from the beam's left end. What do the scales read (F_l and F_r)?

System

L

Block

M •

Beam

m

Scale

Scale

(a)

y

$\vec{F_l}$

$\frac{L}{2}$

$\frac{L}{4}$

$\vec{F_r}$

x

Block

Beam

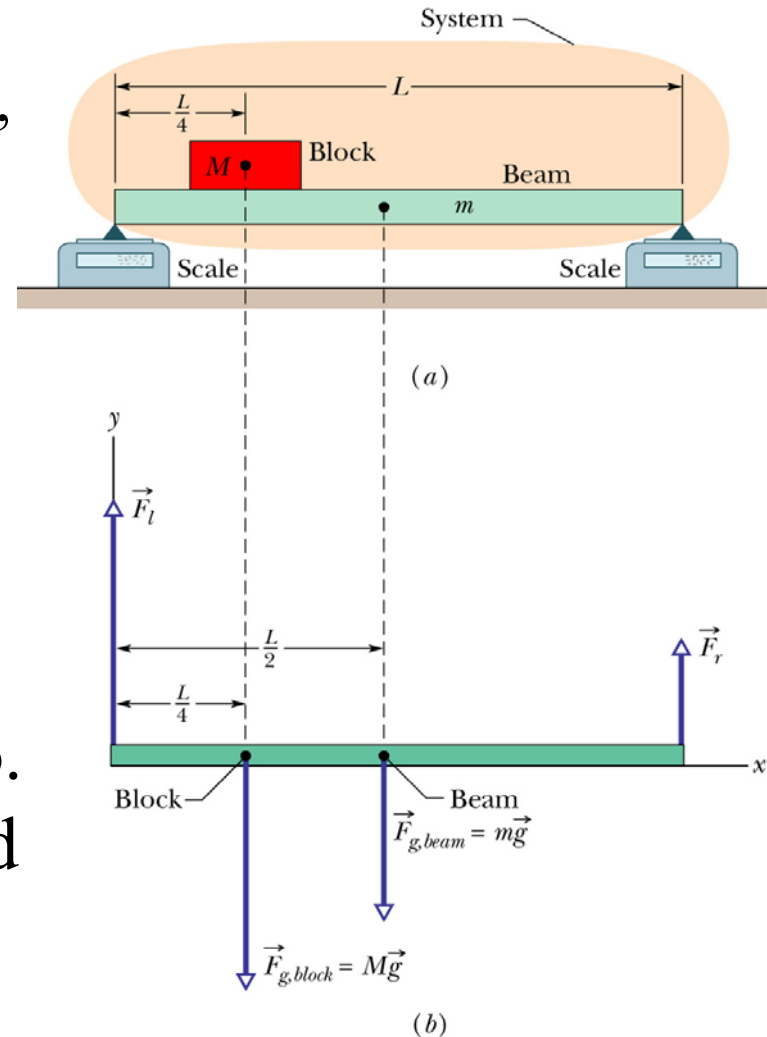$\vec{F}_{g,beam} = m\vec{g}$

$\vec{F}_{g,block} = M\vec{g}$

(b)

Sample Problem 12-1

A beam of length L and mass m = 1.8 kg, is at rest with its ends on two scales. A block of M = 2.7 kg, is at rest on the beam, with its center a distance L/4 from the beam's left end. What do the scales read?

Sum of all the forces on the beam must equal zero.

$$\hat{y}: \quad \sum_i \vec{F}_i = 0 \Rightarrow F_l - Mg - mg + F_r = 0$$

Sample Problem 12-1

A beam of length L and mass m = 1.8 kg, is at rest with its ends on two scales. A block of M = 2.7 kg, is at rest on the beam, with its center a distance L/4 from the beam's left end. What do the scales read?
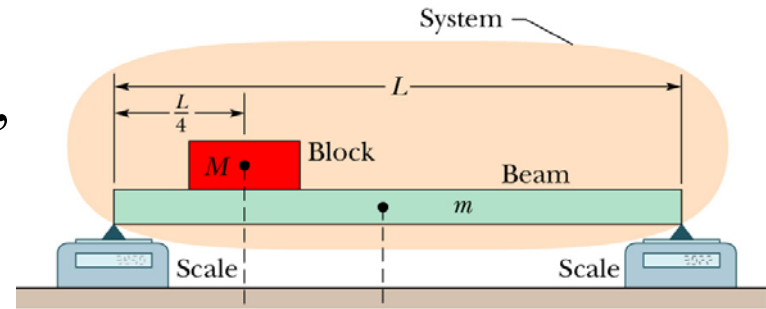

(a)

Sum of all the torques about any point on the beam must equal zero. Choose this point to be the left end of the beam.

(b)

$$\hat{z}: \quad \sum_i \vec{\tau}_i = 0 \Rightarrow (0)F_l - \left(\frac{L}{4}\right)Mg - \left(\frac{L}{2}\right)mg + (L)F_r = 0$$

Sample Problem 12-1

A beam of length L and mass m = 1.8 kg, is at rest with its ends on two scales. A block of M = 2.7 kg, is at rest on the beam, with its center a distance L/4 from the beam's left end. What do the scales read?
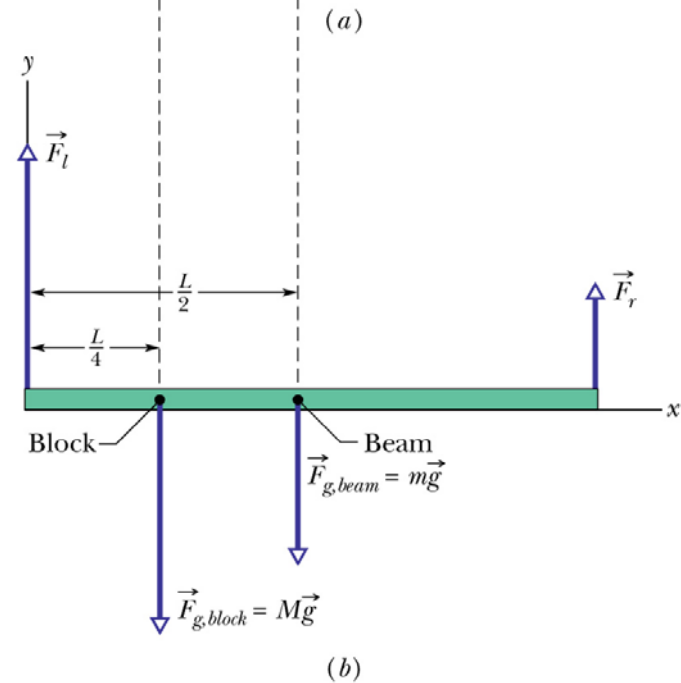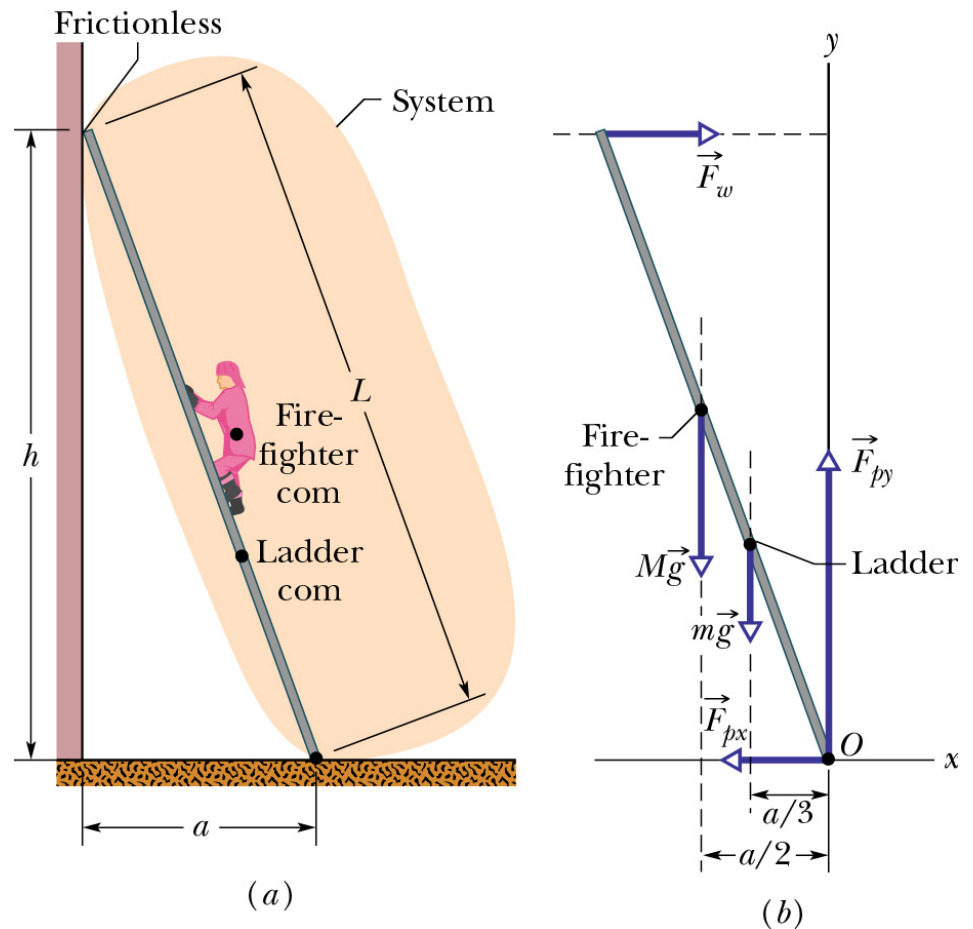

Which yields,

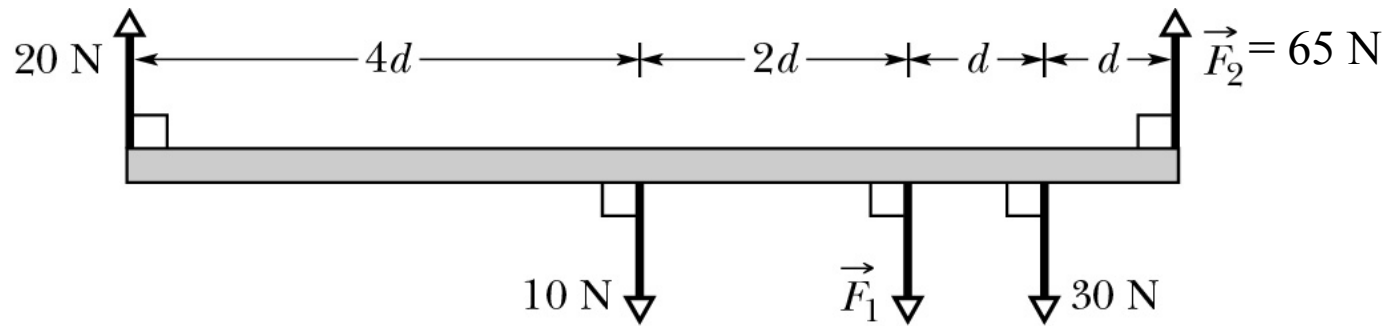$$F_r = \left(\frac{L}{4}\right)Mg + \left(\frac{L}{2}\right)mg$$

and

$$F_l = Mg + mg - F_r = \left(\frac{3L}{4}\right)Mg + \left(\frac{L}{2}\right)mg$$

- Sample problem 12-2: a ladder L = 12 m and mass m = 45 kg leans against a frictionless wall. h = 9.3 m. The ladder's c.o.m. is L/3 from the lower end. A firefighter of M = 72 kg climbs the ladder until her c.o.m. is L/2 from the lower end. What are the magnitudes of the forces on the ladder from the wall and the pavement?
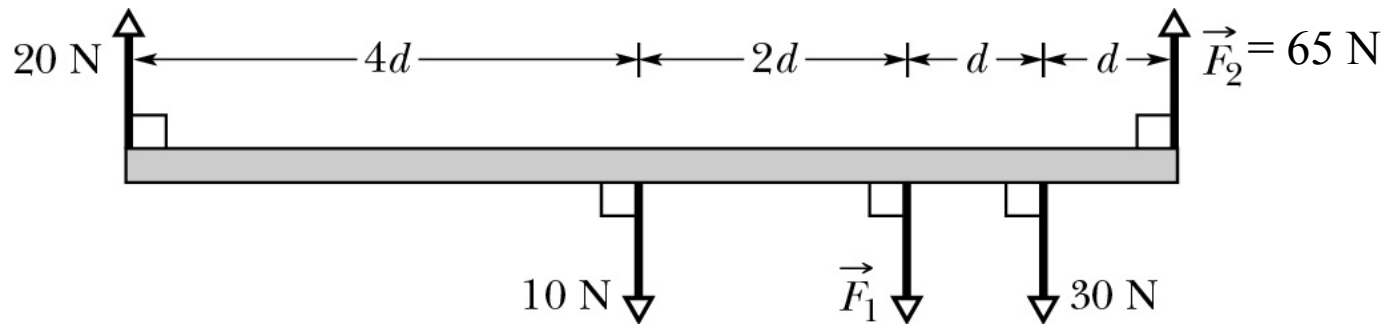

(a)

(b)

Question

20 N ← — 4d — → | ← — 2d — → | ← d → | ← d → $\vec{F_2} = 65$ N

10 N ↓ $\vec{F_1}$ ↓   30 N ↓

What should F_1 be in order to keep the uniform rod in static equilibrium?

Question

$$\hat{y}: \quad \sum_i \vec{F}_i = 0 \Rightarrow 20\,\text{N} - 10\,\text{N} - F_1 - 30\,\text{N} + 65\,\text{N} = 0 \Rightarrow F_1 = 45\,\text{N}$$

$$\hat{z}: \quad \sum_i \vec{\tau}_i = 0$$

$$\Rightarrow -(8d)20\,\text{N} + (4d)10\,\text{N} + (2d)F_1 + (d)30\,\text{N} - (0d)65\,\text{N} = 0 \Rightarrow F_1 = 45\,\text{N}$$
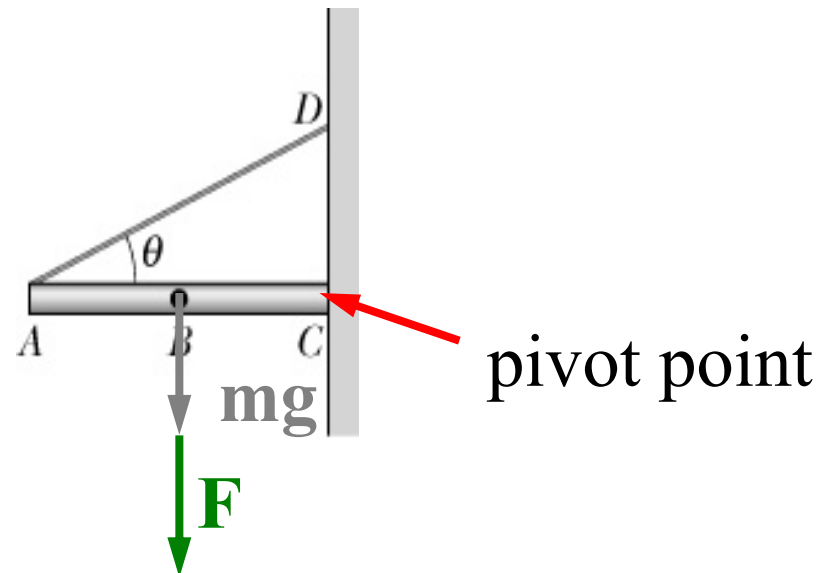

What should F_1 be in order to keep the uniform rod in static equilibrium?

1) 45 N   2) 0 N   3) 40 N

4) it's pointed in the wrong direction   5) none of the above

A Quiz

Suppose that the (uniform) horizontal rod is held just barely in static equilibrium by the rope and the pivot point. The grey arrow is the weight of the rod at the center of mass and the green arrow is a movable applied downward force. What happens when F is moved toward A?
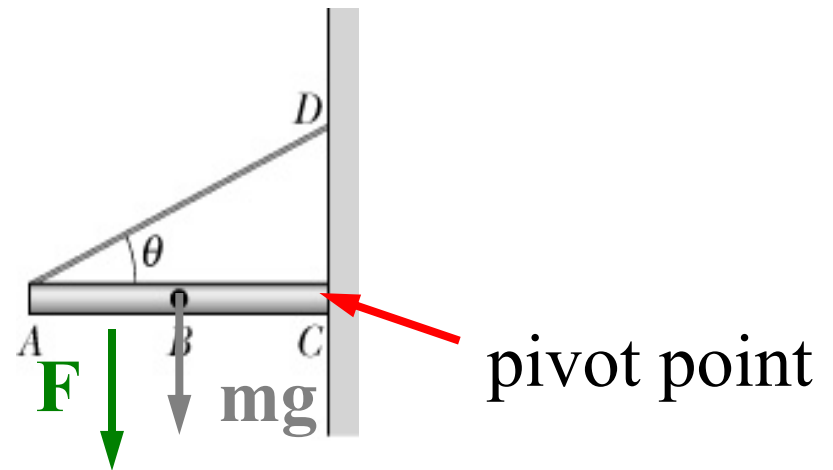
1) nothing
2) the net force increases
3) the net torque increases
4) the net torque decreases
5) none of the above

pivot point

A Quiz

Suppose that the (uniform) horizontal rod is held just barely in static equilibrium by the rope and the pivot point. The grey arrow is the weight of the rod at the center of mass and the green arrow is a movable applied downward force. What happens when F is moved toward A?

1) nothing
2) the net force increases
3) the net torque increases
4) the net torque decreases
5) none of the above

A Quiz

$$\hat{y}: \quad \sum_i \vec{F}_i = F_{pivot,y} - mg - F + T\sin\theta$$

$$\hat{x}: \quad \sum_i \vec{F}_i = -F_{pivot,x} + T\cos\theta$$

$$\sum_i \vec{\tau}_i = lF + \frac{L}{2}mg - LT\sin\theta \quad > \quad \frac{L}{2}F + \frac{L}{2}mg - LT\sin\theta$$

since $l > L/2$, the torque increases

1) nothing
2) the net force increases
3) the net torque increases
4) the net torque decreases
5) none of the above

pivot point
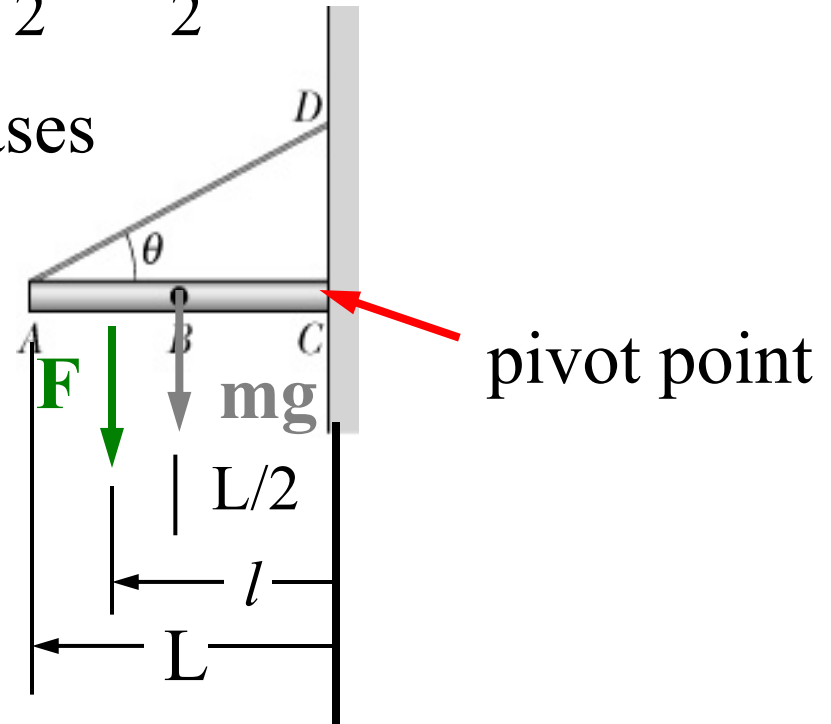
Stress and Strain

Microscopic view of materials: Materials are made of atoms held in place by electrostatic interactions with neighboring atoms.

These interactions are such that the atoms are constantly in harmonic motion about their equilibrium positions.
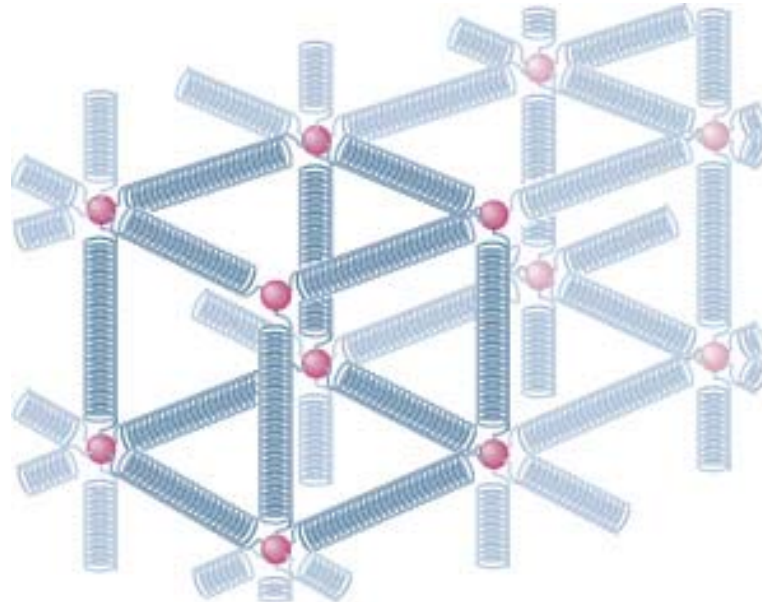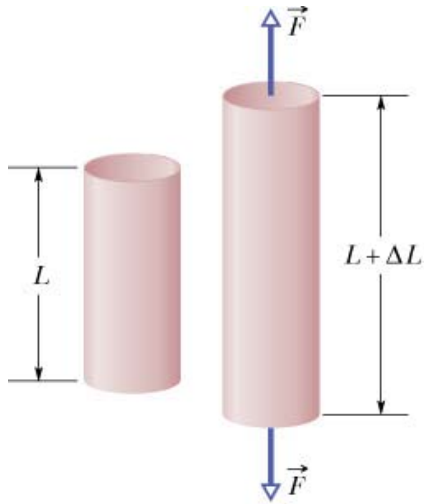
Stress and Strain

External forces can be exerted on these atoms. The atoms will react to these forces depending on their microscopic environment.
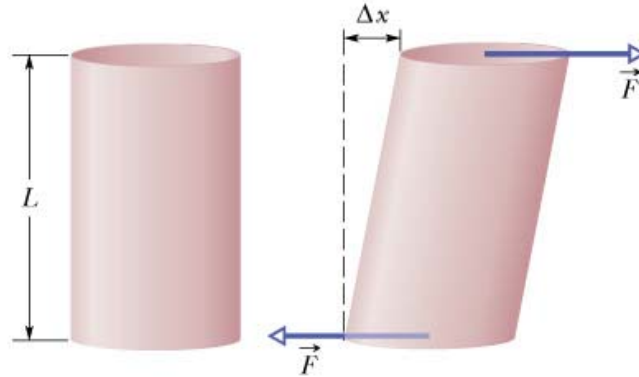
F

F

F

F
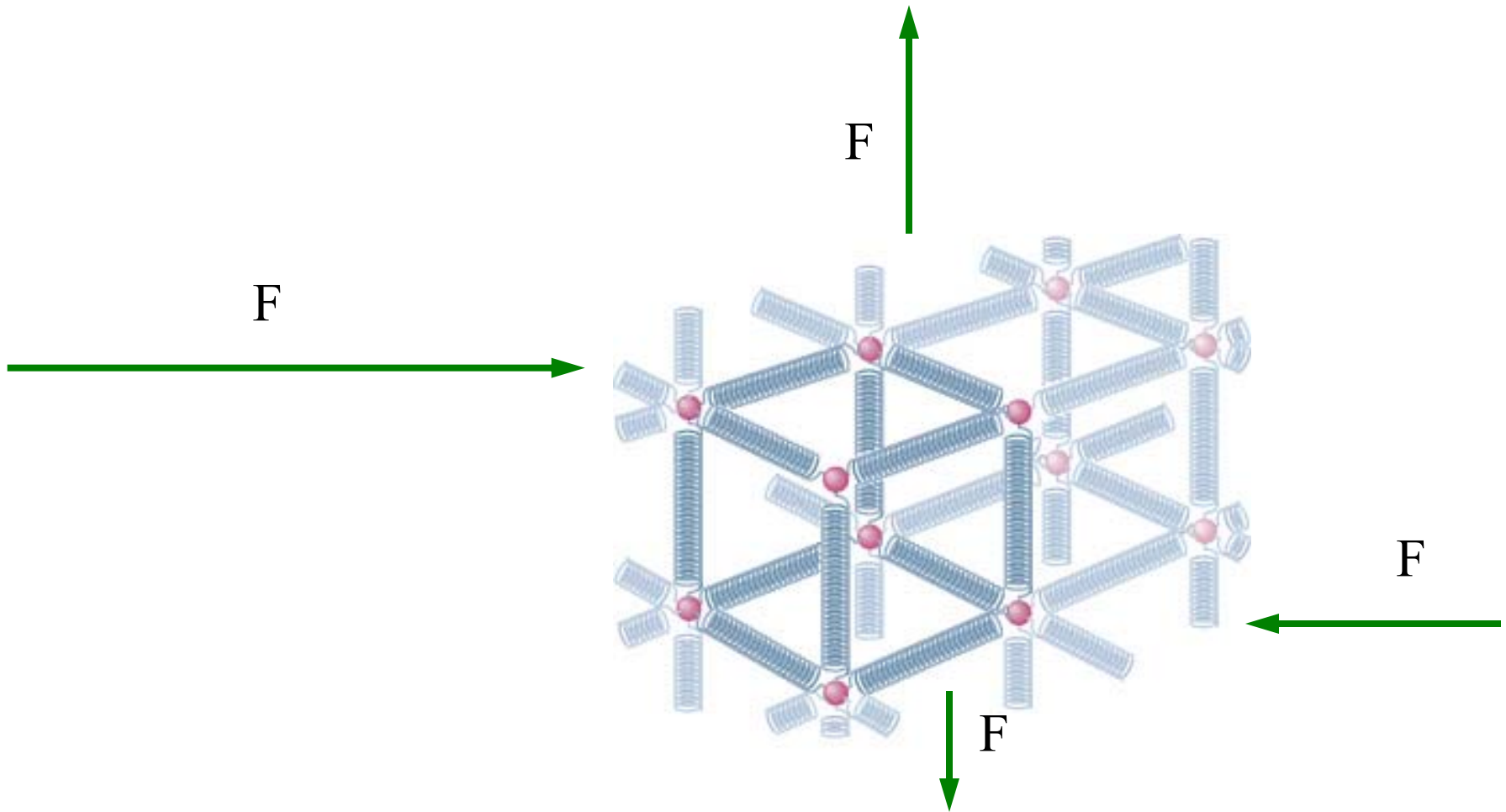
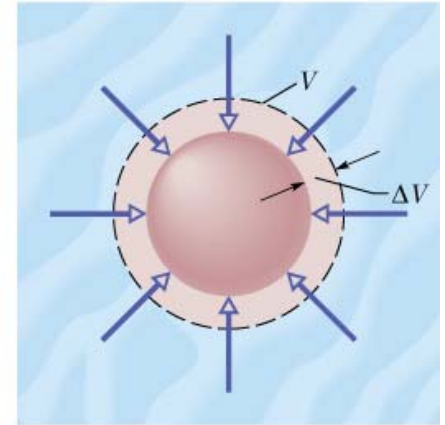# Deformation Types

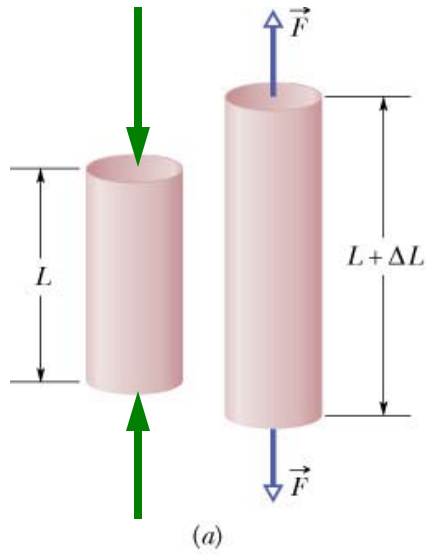elongation
(tensile strength)

shear

hydraulic
compression

F

$L + \Delta L$

L

F

Δx

F

L

F

V

ΔV

(a)

(b)

(c)

modulus is a constant depended on how much the materials react (deform) to the applied forces.

stress = modulus x strain

Elongation (tensile strength) and compression

Stress is defined as perpendicular force per unit area. **Stress = F/A**

Strain is defined as the fractional change in the length of the object. **Strain = ΔL/L**

Young's modulus E is the proportionality constant.
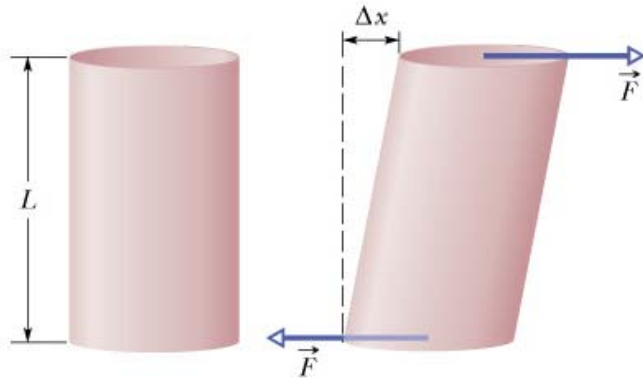
$$\frac{F}{A} = E\frac{\Delta L}{L}$$

Shear stress

Stress is defined as force in the plane per unit area. **Stress = F/A**

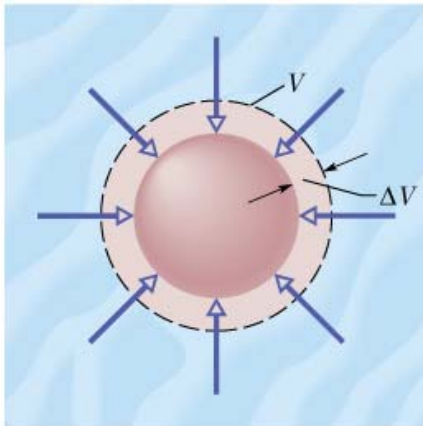Strain is defined as the fractional change in the movement of the object.
Strain = Δx/L

Shear modulus G is the proportionality constant.

$$\frac{F}{A} = G\,\frac{\Delta x}{L}$$

Hydraulic Stress

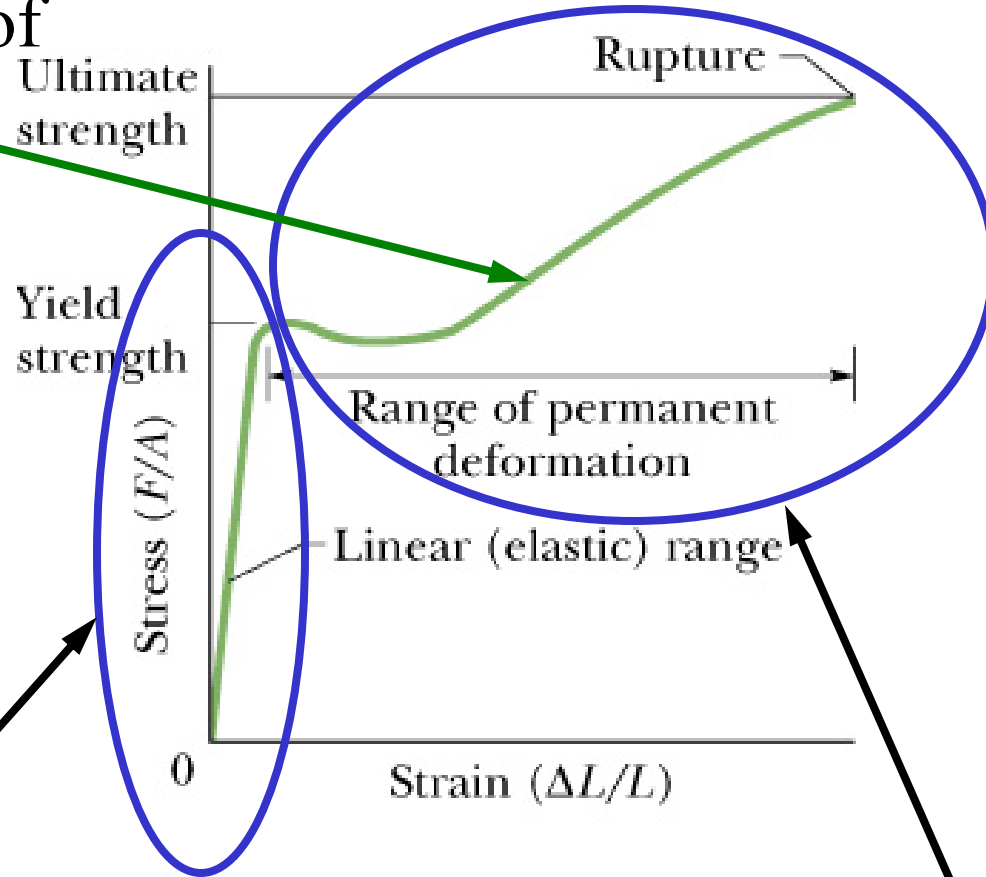For three dimensions we use pressure, which is also defined as force per unit area. **Pressure p = F/A**

Strain is defined as the fractional change in the volume of the object. **Strain = ΔV/V**

Bulk modulus B is the proportionality constant.

$$p = \frac{F}{A} = B\frac{\Delta V}{V}$$

Materials science studies cause of specific shape of this curve.

Elastic Limit

Region we'll be discussing this semester.

Additional region of interest to engineers.